# THE ELEMENTS OF REINFORCED CONCRETE DESIGN

A COMPLETE LIST OF
"CONCRETE SERIES" BOOKS
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# THE ELEMENTS OF REINFORCED CONCRETE DESIGN

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# PREFACE TO FIRST EDITION

IN the preparation of this book the keynote of the author's efforts has been simplicity in both presentation and method, for it is felt that text-books are of most value to students if they keep to essentials and present them as simply as possible. This can be done without encouraging the student not to think for himself, in fact it leaves him free to think and encourages him to use his own judgment.

Complicated proofs have been avoided, it being considered much more important that the student should understand the physical implications of the statements made; while, therefore, many statements are axiomatic, others will appear perfectly reasonable, and a few may remain for the proofs to be investigated, should this become necessary, at a later stage.

An effort has been made to set down the working of examples clearly and simply so that the reader may concentrate on the method, for an appreciation of the method is of most importance, and therefore round numbers have so far as possible been used; there is no need to make this work an exercise in arithmetic.

In design it becomes necessary to make numerous assumptions on which to build deductions and so obtain results. Actual conditions may interfere considerably with these assumptions, and it is in our judgment of these that experience plays so important a part. Many assumptions which, strictly speaking, are untrue are made in the development of the principles of reinforced concrete design: thus plastic yield may interfere very much with the assumed relationship of stress to strain. However, this is not serious, and it may be taken that the methods explained in the following pages will, if properly applied, result in safe and economical design.

In the design of complicated structures as a whole individual assumptions will have to be made which cannot be brought to the touchstone of general usage. In such cases individual judgment must be used, and care and imagination must be exercised in the consideration of possible variations in the finished structure from the conditions assumed in design.

There is a further snare in the path of the unwary. As his experience increases the designer will embark upon new methods of dealing with complicated design. The most complicated method will not always yield the surest result. There is always a danger of being lulled into a false sense of security by abstruse mathematics—there is no magic charm in complicated and intricate computation; there is indeed some danger, and the safest guide in most matters is to cling fast to common sense.

One word about the use of design charts. Until he has understood the methods and working which these are intended to replace the student has no

business with them; they are useless in the learning stage, and useful only when applying knowledge to the solution of design problems. We must seek always first to understand, secondly to think constructively, and finally, when we have so to speak "found our feet," to use simple and direct methods of easing the tedious work of arithmetical solution.

Reinforced concrete design is to those who pursue it from choice a fascinating study. It is a young science, and we have still much to learn. The student embarking upon his first studies in this subject will soon realise that he will never exhaust them, and that he and his fellows may themselves have a part to play in the advancement of knowledge in their chosen profession.

The author is indebted to Mr. C. S. Chettoe, B.Sc., M.Inst.C.E., with whom as joint author he was associated in the production of "Reinforced Concrete Bridge Design," and to the publishers, Messrs. Chapman & Hall, Ltd., for permission to use certain drawings and design charts prepared for that work; also to Mr. J. E. Jones, M.Sc., A.M.Inst.C.E., for reading the text and making several suggestions.

H. C. A.

LONDON, 1933.

## PREFACE TO THIRD EDITION

SINCE the second edition of this book was published higher concrete stresses have been widely adopted. Those in current use for highway bridge design are given in Memorandum No. 577 of the Ministry of Transport (Memorandum on Bridge Design and Construction), published by His Majesty's Stationery Office, and are broadly in line with the recommendations of the Code of Practice for the Use of Reinforced Concrete in Buildings issued by the Department of Scientific and Industrial Research in 1934.

It has not been thought necessary to re-write the book to conform to the new stresses; method of design is independent of arithmetic, and as the methods remain unchanged the student will find no difficulty in working out the examples to other stresses. A small addition has been made to the section dealing with reinforcement to bring this up to date.

In view of the progress which has been made in prestressed concrete the student should not now be left without some knowledge of the underlying principles, and a short chapter has been added to supply this want.

H. C. A.

1947.

# CONTENTS



## TABLES

# NOTATION

| | | |
|---|---|---|
| $p$ | (see p. 8) | Fibre stress |
| | ( ,, 40) | Steel ratio, $\frac{A_s}{bd}$. |
| | ( ,, 105) | Bearing pressure intensity. |
| $b$ | ( ,, 39) | Breadth of section. |
| $t$ | ( ,, 53) | Slab thickness. |
| $E$ | ( ,, 8) | Modulus of elasticity. |
| $E_c$ | ( ,, 37) | ,, ,, ,, for concrete. |
| $E_s$ | ( ,, 37) | ,, ,, ,, ,, steel. |
| $M$ | ( ,, 8) | Applied bending moment. |
| $M_1$ | ( ,, 84) | ,, ,, ,, in doubly reinforced beam. |
| $y$ | ( ,, 9) | Distance of (extreme) fibres from neutral axis. |
| $R$ | ( ,, 9) | Radius of curvature in bending. |
| $I$ | ( ,, 10) | Moment of inertia of section. |
| $I_c$ | ( ,, 96) | ,, ,, ,, ,, concrete in section. |
| $I_s$ | ( ,, 96) | ,, ,, ,, ,, steel in section. |
| $Z$ | ( ,, 11) | Section modulus $\left(=\frac{I}{y}\right)$. |
| $f_c$ | ( ,, 39) | Concrete fibre stress (compression). |
| $f_s$ | ( ,, 39) | Steel ,, ,, (tension). |
| $f_s'$ | ( ,, 84) | ,, ,, ,, (compression). |
| $A_s$ | ( ,, 39) | Sectional area of steel. |
| $A_t$ | ( ,, 84) | "Additional" area of tensile steel in doubly reinforced beam. |
| $A_c$ | ( ,, 84) | Area of compressive steel in doubly reinforced beam. |
| $d$ | ( ,, 39) | Effective depth of section (depth to centre of tensile steel from compression face). |
| | ( ,, 51) | Bar diameter (when referred to as such). |
| | ( ,, 84) | Embedment of reinforcing bar. |
| $n$ | ( ,, 37) | Modular ratio $\left(\frac{E_s}{E_c}\right)$. |
| $k$ | ( ,, 39) | Coefficient of $d$ (neutral axis to compression face). |
| | ( ,, 99) | Coefficient of $h$, for combined bending and compression equations. |
| $k_1$, $k_2$, etc., | | Coefficients of $l$ (span length). |
| $M_x$ | | Bending moment at section distant $x$ units from support. |
| $R_A$, $R_B$, etc., | | Reactions at supports $A$ and $B$, etc. |
| *D.L.* | | Dead load. |
| *L.L.* | | Live load. |
| $j$ | (see p. 39) | Coefficient of $d$ (centre of compression to centre of tension). |
| $R$ | ( ,, 40) | A constant representing $f_s \times p \times j$. |
| $\Sigma_o$ | ( ,, 42) | Sum of perimeters of reinforcing bars per foot width of slab, or per group. |
| $V$ | ( ,, 11) | Total shear on section. |
| $V_s$ | ( ,, 48) | ,, ,, taken by web reinforcement. |
| $V_1$ | ( ,, 49) | Shear taken by bent-up bars, in presence of stirrups. |
| | ( ,, 107) | A particular shear value. |
| $v$ | ( ,, 13, 46) | Maximum unit shear on section (disregarding web reinforcement). |
| $s$ | ( ,, 48) | Stirrup spacing. |
| | ( ,, 105) | Unit punching shear. |
| $u$ | ( ,, 50) | Unit bond stress on reinforcing bars. |
| $e$ | ( ,, 96) | Eccentricity of thrust. |

# CHAPTER I

## THE SIMPLE BEAM: BENDING AND SHEAR

### The Method by which a Beam Supports a Load.

WHEN a block of wood rests on a table the pressure exerted by each on the other where their surfaces are in contact is equal to the weight of the block. If a load $W$ is placed on the top of the block the pressure is increased by the amount of this load. If instead of one table the block rests on two tables placed together, half of the block resting on each, the total pressure is still as before, but one half of the support comes from each table. If the tables are moved apart so that the block is just supported on them equally as shown in *Fig.* 1, the total pressure acting through each support is unchanged.

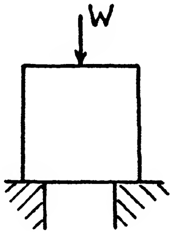



Fig. 1.

The block may be replaced by a plank of wood or a beam (*Fig.* 2) and the same forces will act. It will be realised, however, that as the tables or supports are moved apart there is a tendency for the block or the beam to sag under its own weight and the loads carried by it. There was no such tendency when

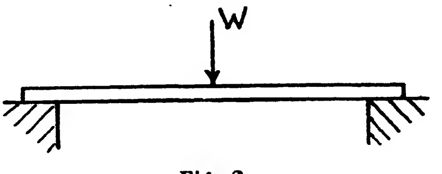
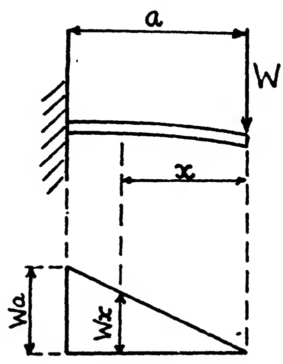



Fig. 2.

the support was immediately below the block and its load, but bending is introduced as soon as the downward forces (the weight of the block or beam, and the load $W$) are brought out of line with the opposite or balancing forces acting through the supports. This bending causes the beam to assume a curved shape.

The amount of bending is proportional to the weight, load, or forces acting, and the extent to which these are out of line.

Thus in the case of a beam fixed at one end in a wall (forming what is termed a cantilever) and having a weight $W$ hung at one end (*Fig.* 3), the amount of bending at the socket (wall face) is proportional to $W$ and $a$. These two quantities are multiplied together and the product $Wa$ is termed the bending moment, and the curvature in the beam at the wall will be proportional to this value. Half-way along the cantilever the bending moment will be $W\frac{a}{2}$, and at any distance $x$ from $W$ the bending moment will be $Wx$.




B. M. D.

**Fig. 3.**

If we wish to make a drawing to show by the vertical depth of the diagram at any section the size of the bending moment, we should have in this case a triangle as shown in the bottom part of *Fig.* 3. This is termed the bending moment diagram (B.M.D.).

Turning our attention again to the beam in *Fig.* 2, if the load $W$ is moved to one end over the support the balancing force (termed the reaction) developed below will occur wholly in the one support, but as the load $W$ is moved away

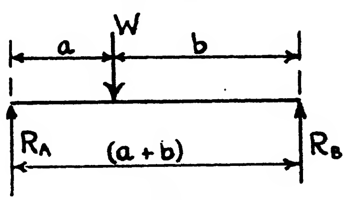



**Fig. 4.**

from the end the far support shares more and more in sustaining it, until it in turn carries the whole of the load when the weight is immediately above it. The reactions developed in the two supports are thus directly dependent upon the position of the load: in *Fig.* 4, with $W$ placed at any position in the span, $R_A = W \times \frac{b}{(a+b)}$ and $R_B = W \times \frac{a}{(a+b)}$, as will be explained presently.

The moment of a force about a point is its turning effect, or leverage effect, and its value is equal to the force itself multiplied by the perpendicular distance from the point (about which the moment is taken) to the line of the force.

The bending moment is the sum of the external or applied turning effects or leverage effects of all forces, and may be calculated for any section in a beam by adding together algebraically (that is, adding those moments which tend to turn in one direction, and subtracting those operating in the reverse direction) all the turning forces to one side of the section. This will give the bending moment at the section. If this sum were the only turning force acting on the section movement would take place so that the section spun round, but as the beam remains in position (or in equilibrium as it is called) we know that an equal balancing force must exist within the section (internal force) operating in the opposite direction ; this is developed by the outside (or external) forces operating on the beam on the other side of the section. These internal forces will be referred to again later as it is the determination of these forces and the provision for them by way of concrete and reinforcement which is the main work of design.

Given the freely supported beam in *Fig.* 4 we can determine the values of the two reactions $R_A$ and $R_B$ because of two simple facts.

Because the beam moves bodily neither up nor down but is in equilibrium we know that the sum of $R_A$ and $R_B$ is equal to $W$, or writing this as an equation,

$$R_A + R_B = W \qquad (1)$$

Because the beam does not rotate but is in equilibrium we know that the algebraic sum of the bending moments (both external and internal) at any section is zero. The internal moments are not yet known so we select a section where there are no more forces to one side and where there can therefore be no internal moment. This is at $R_B$ (or $R_A$, whichever we like). The reaction $R_B$ passes through this position, and having no leverage or turning effect it cannot produce a bending moment on this section.

Taking moments about a section above $R_B$ we find a clockwise moment $R_A(a + b)$ and an anti-clockwise moment $Wb$, and writing this as an equation :

$$R_A(a + b) - Wb = \text{bending moment about } R_B = 0 \qquad (2)$$

This gives us the value of

$$R_A = \frac{Wb}{(a + b)} \qquad (3)$$

By substituting this value for $R_A$ in equation (1) we have

$$R_B = (W - R_A) \qquad (4)$$

Under ordinary conditions the beam is horizontal and the forces acting on it are vertical. Where these forces are inclined we resolve them vertically, or where the beam is inclined we resolve all forces at right-angles (or normal) to the beam ; and because the complete system is in equilibrium we equate their algebraic sum to zero. When there are several forces similar to $W$ we use a general equation.

Resolving vertically,

$$R_A + R_B - \Sigma W = 0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This corresponds to equation (1). The symbol $\Sigma$ is the Greek letter sigma which is the initial letter of sum, and $\Sigma W$ means the sum of all forces similar to $W$.

Similarly we take the sum of the moments of all external forces to one side of a free support, and because of equilibrium equate it to zero.

Taking moments about $B$ and writing the result as an equation:

$$R_A(a + b) - \Sigma Wx = 0 \quad . \quad . \quad . \quad . \quad (6)$$

$\Sigma Wx$ means the sum of all moments similar to $Wx$. The distance is denoted by the variable $x$ as being general, whereas $b$ is a particular dimension having a special value.

EXAMPLE.—A freely-supported beam carries loads of 12 tons, 10 tons and 8 tons in the positions shown in *Fig.* 5. Determine the reactions $R_A$ and $R_B$.

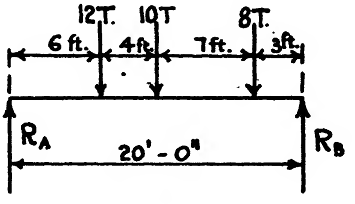



Fig. 5.

Resolving vertically,

$$R_A + R_B - 12 - 10 - 8 = 0.$$

Taking moments about $B$,

$$R_A \times 20 - 12 \times 14 - 10 \times 10 - 8 \times 3 = 0.$$

$$\therefore R_A = \frac{168 + 100 + 24}{20}$$

$$= 14{\cdot}6 \text{ tons,}$$

$$\text{and } R_B = 30 - 14{\cdot}6$$

$$= 15{\cdot}4 \text{ tons.}$$

## Shear.

Referring again to *Fig.* 4, it is clear that if we resolve vertically the external forces to the left of any section between $R_A$ and $W$ we get the value $R_A$. Because of equilibrium this must be balanced by an internal force or shear in the same way that we saw there were internal moments to balance the external ones. If we consider the section $XX$ in *Fig.* 6, the portion of beam to the left is tending to slide up due to the force $R_A$ and the portion of beam to the right is tending to slide down due to the force $W$. If the beam were sawn through and the forces $R_A$ and $W$ could still be made to operate this relative movement would take place. The resistance to this internal shear is prevented by the cohesion

of the beam. The tendency for the faces to slide one over the other is due to the shearing force. To the left of $W$ the shearing force is equal to $R_A$; to the right of $W$ it is equal to $R_B$. The change over is due to $W$ and occurs at the section where $W$ operates.

## Shear Force and Bending Moment Diagrams.

The forces just referred to may be represented diagrammatically as shown by the shear force diagram (S.F.D.) in *Fig.* 6. Where several loads occur on the span the method is exactly the same; a change in the value occurs at each section where a load is applied, and the shears at the various sections are

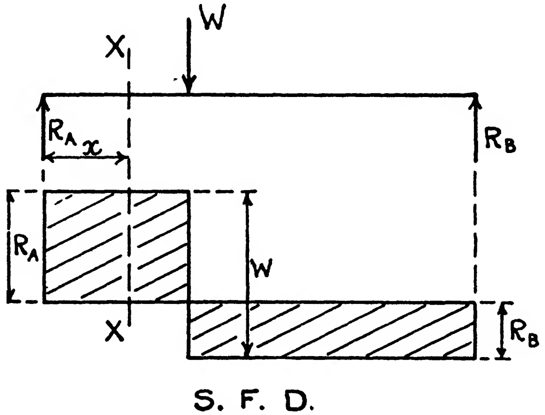



Fig. 6.

evaluated by resolving vertically all the external loads to one side of the section and taking their algebraic sum. Positive values are plotted on one side of the base line and negative values on the other.

A positive ordinate in the shear force diagram is usually taken to indicate that the portion of beam on the left-hand side of the section in question would, if free to move, slide upwards relative to the right-hand portion of the beam. The sign is purely conventional.

When the loading is not concentrated at points but is distributed, the shear clearly changes gradually. If the load is uniformly distributed the change occurs at a regular rate as in *Fig.* 7. Concentrations and a distributed load when combined give a shear force diagram as in *Fig.* 8.

Since the bending moment at any section is equal to the sum of the external forces to one side of the section multiplied by their lever arms, it follows that the bending moment at any section is equal to the area of the shear force diagram from the beginning of the beam up to that section. This is easily seen in the case of *Fig.* 6, where the bending moment at section $XX$ is $R_A x$; it is true for all loadings. When the shear changes sign the area must be considered algebraically. The accumulated area of the shear force diagram taken up to any point in the beam is thus represented by the vertical depth of the bending moment

diagram at that point or section. With a uniform loading as in *Fig.* 7 the bending moment diagram becomes a parabola.

In a freely-supported beam loaded in one direction (downwards, for example) the bending moment will always be of the same sign. Sagging bending moments are generally regarded as positive, and hogging moments, such as in the case

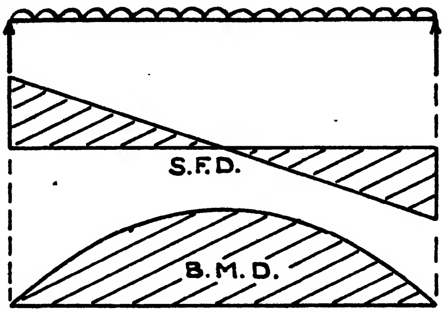



Fig. 7.

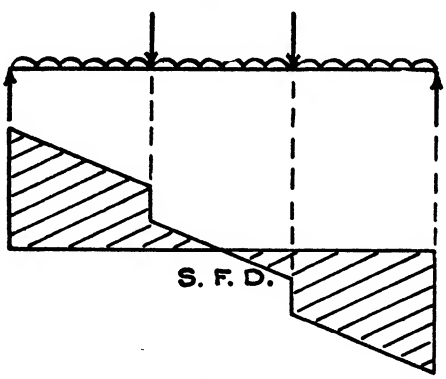
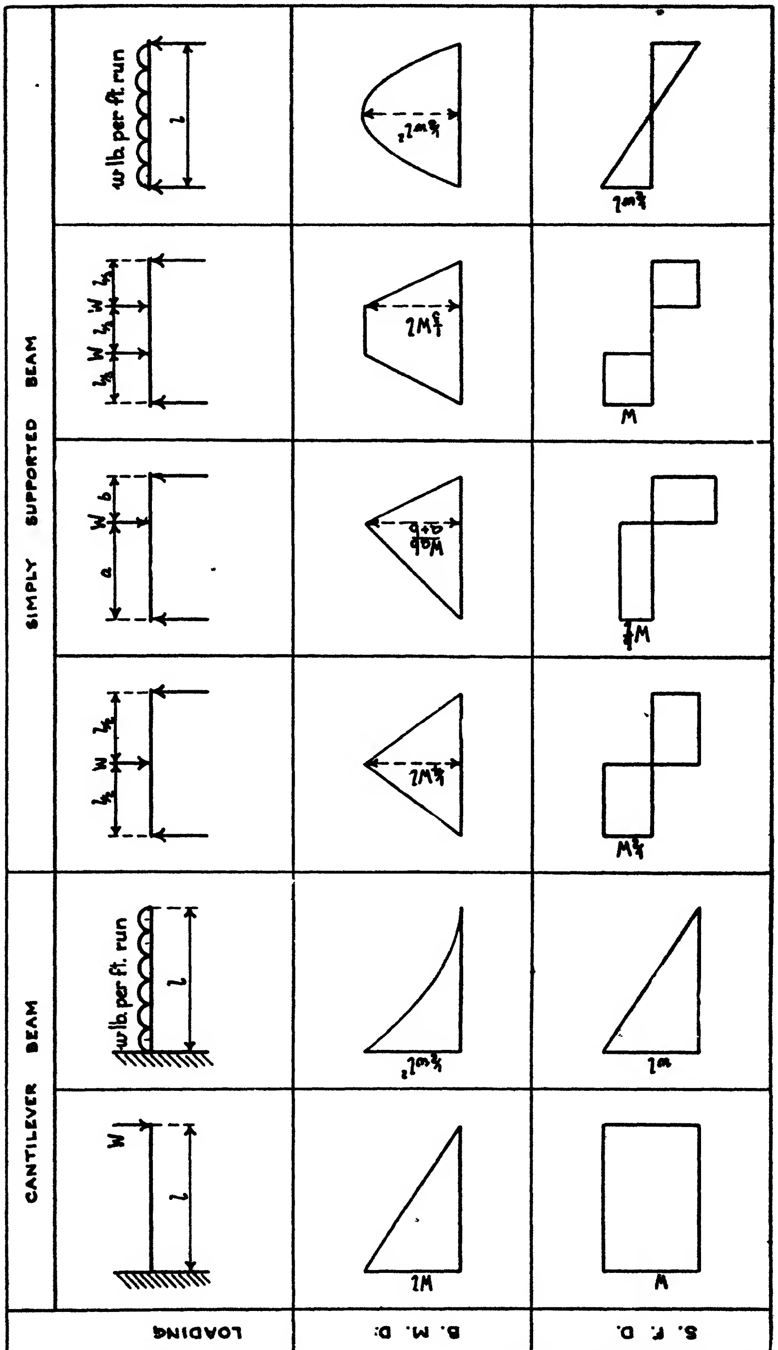



Fig. 8.

of *Fig.* 3 and in continuous beams over the supports, are considered to be negative. Typical loadings with their bending moment and shear force diagrams are given in *Fig.* 9.

### Internal Forces (Beam Stresses) and Moment of Resistance.

The external or applied forces develop bending moments which are independent of the internal structure of the beam whether this is composed of timber, steel, or reinforced concrete. If no internal forces were developed to resist the applied bending moments and shears the beam would collapse, but if the beam is stiff enough it will merely bend and offer the necessary resistance. The internal stresses depend on the size and shape of the beam cross section; in a beam composed of more than one material the stresses depend upon other special characteristics which will be explained later.



SIMPLY SUPPORTED BEAM
CANTILEVER BEAM
LOADING
B. M. D.
S. F. D.
w lb. per ft. run
w lb. per ft. run


Fig. 9.

The deformation or degree of bending which results from the application of the external forces depends upon the internal stresses (and thus upon those factors which, as enumerated already, influence these stresses) and also upon the elasticity of the material of the beam, or the relationship of strain to stress. Stress * at any point is the load per unit area of the cross section at that point; strain is the consequent elongation or contraction per unit of length. Hooke's Law states that within limits strain is proportional to the stress from which it results.

It is unnecessary at this stage to introduce the complications of awkward beam cross sections or of bending moment varying from section to section; the principle once understood is easily applicable to other cases. For this reason we select now a simple beam of rectangular cross section and apply to it a constant bending moment—that is, a moment which we will call $M$ is applied to each end and no loads are applied to the beam.

The following symbols will be used:

$p$ = the unit stress (stress per unit of area).

$E$ = Young's modulus, or modulus of elasticity for tension and compression, and is the ratio of stress to strain. Its value for any material is thus the unit stress required to produce unit elongation in unit length (that is, to double the original length).

$M$ = the applied bending moment.

If a stick of india-rubber is bent by the fingers, it will be noticed that in taking up curvature the face on the outside of the bend is stretched or elongated and the face on the inside of the bend is compressed or contracted. Exactly the same happens with other materials, although with a more rigid stick such as a bar of iron or steel the magnitude of the deformation is so small as not to be noticeable to the naked eye. The difference in degree between the two deformations is dependent upon the value of $E$, which for steel is about 30,000,000 lb. per square inch, and for good india-rubber is between 300 and 400 lb. per square inch.

For all practical considerations the stretching on the outside of the bend falls off at a regular rate towards the centre line † of the beam cross section; passing this line it changes over to compression which increases at a regular rate until the maximum contraction is reached at the inside face of the bend. The extreme faces where the maximum deformations occur are termed the extreme fibres, and stress at any point in the beam section is referred to as fibre stress.

Having applied a constant moment to our beam the curvature also will remain constant and the beam will therefore form a portion of a circle as shown in *Fig.* 10. In the cross section $XX$ the line $NA$ where the change over from elongation to contraction occurs denotes those fibres whose length is unchanged by the process of bending; it is termed the neutral axis. In the main diagram (*Fig.* 10) this position in the beam is represented by the circumferential line of the circle and is termed the neutral surface or plane. It has a constant radius

* The term total stress is sometimes used to denote stress over an area, in which case stress (that is, per unit of area) is termed unit stress.

† A line through the centre of gravity of the section. This is half-way down in a plain rectangular section, but is not so, unless by coincidence, in a section of irregular shape.

$R$. When the applied moment changes from section to section, as in a loaded beam, $R$ also varies from section to section, but the relationships to be explained later will still apply to any section of a beam, the stresses varying with the moment.

Consider *Fig.* 10. The two ends of the beam which originally were parallel when the beam was straight are now radial. The neutral plane of the beam, which is the same length as before bending, forms a portion of the circumference of the circle and its length can be represented by the term $C \times 2\pi R$, where $C$ is a constant. The new lengths of the extreme fibres, which originally were equal to that of the neutral plane, are therefore represented by $C \times 2\pi(R + y)$ on the outside and $C \times 2\pi(R - y)$ on the inside of the curve, where $y$ is the distance from the neutral plane to the extreme fibre (in this case one-half the

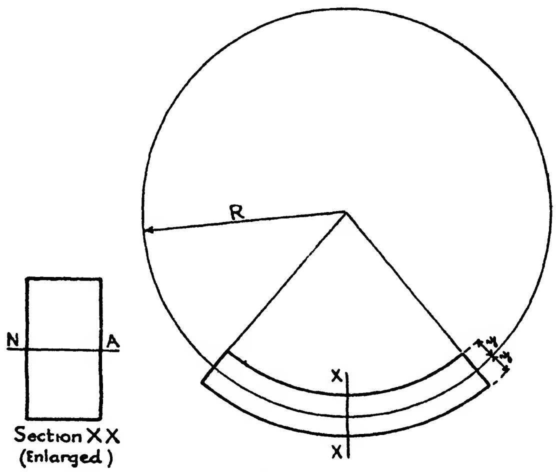



Fig. 10.

depth of the beam). We see that the extreme fibres have stretched so that the ratio of new length to original length is

$$\frac{C \times 2\pi(R + y)}{C \times 2\pi R} = \frac{R + y}{R}.$$

In a length $R$ the increase is $y$, or in unit length the strain is $\frac{y}{R}$. By our definition of $E$ $\left(= \frac{\text{stress}}{\text{strain}}\right)$ we now see that

$$p = \frac{y}{R} \times E \text{ or } \frac{p}{y} = \frac{E}{R}. \qquad . \quad . \quad . \quad . \quad . \quad (7)$$

This relationship holds for any value of $y$ between the neutral axis and the extreme fibre, and does not depend upon the shape of the section. The stress is compression on one side of the neutral axis and tension on the other.

We turn our attention now to the internal moment or moment of resistance of the beam, which in order to maintain equilibrium must be equal to the external or applied bending moment at the section. *Fig.* 11 reproduces section $XX$ from *Fig.* 10 and shows also the stress diagram in which the stress intensity is proportional to the distance from the neutral axis. Consider the small strip shown cross hatched, of area $a$ sustaining a stress of intensity $p$, and distant $y$ from the neutral axis. As $p = \frac{yE}{R}$ the total stress on area $a$ is $ay \times \frac{E}{R}$, and its moment or leverage effect about the neutral axis is $ay^2 \times \frac{E}{R}$. The moments on a sectional face due to compression above the neutral axis and tension below the neutral axis are both of the same algebraic sign, and the sum of all moments

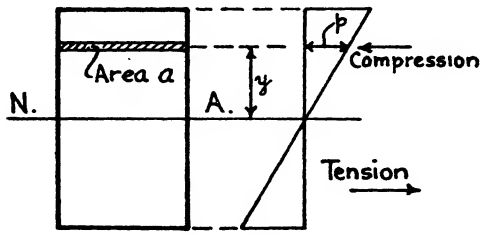



Fig. 11.

in the section similar to the above term, which sum is represented by the expression $\Sigma ay^2 \times \frac{E}{R}$, is equal to the applied bending moment $M$. As $E$ and $R$ are constants these may be taken outside the summation symbol, and we have

$$\frac{E}{R} \times \Sigma ay^2 = M \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The expression $\Sigma ay^2$ is a property of the section; it is called the moment of inertia * of the section and is represented by the symbol $I$. Values of the moment of inertia for ordinary sections are given in almost any engineering handbook. For a rectangular section $I = \frac{bh^3}{12}$ where $b$ is the breadth and $h$ the depth of the section.

Equation (8) may now be re-written

$$\frac{E}{R} = \frac{M}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8\text{A})$$

and the complete relationship (see equation (7)) is thus:

$$\frac{p}{y} = \frac{E}{R} = \frac{M}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

* We refer to an area as having two dimensions, that is, sq. in. or in.$^2$ In the same way $I$, which equals $\Sigma ay^2$, has four dimensions (in.$^4$). The section modulus $\frac{I}{y}$ therefore has three dimensions (in.$^3$).

The term $\frac{I}{y}$ is also a property of the section and is termed the section modulus, represented by $Z$, so that

$$p = \frac{My}{I} = \frac{M}{Z} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

### Internal Shear Stress in a Loaded Beam.

The method of obtaining the external shear has already been explained. The shear at any section may be designated by the symbol $V$. The total internal vertical shear developed by the cohesion of the material comprising the beam must be equal to the external shear at the section under consideration. It is not uniformly distributed over the whole section.

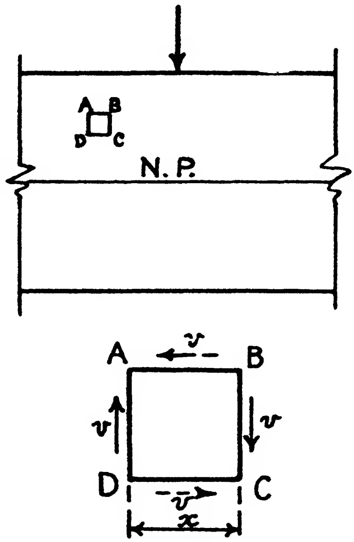



**Fig. 12.**

The vertical shear intensity at any point in the section is equal to the horizontal shear at that point. This may be seen by considering the forces acting on a very small cube $ABCD$ inside the beam as shown in *Fig.* 12. The cube is taken at any point not immediately beneath a load, and is so small that the shear does not change measurably between faces $AD$ and $BC$ (the weight of the beam itself being the only factor contributing to such a change). The face dimension of the cube may be taken as $x$, and the shear intensity on faces $AD$ and $BC$ is denoted by $v$ (lb. per square inch), the directions being as shown by the arrows; these forces are, of course, supplied by the adjacent portions of the beam. The longitudinal compression indicated in *Fig.* 11 need not here be considered as it operates on both vertical faces (merely squeezing the cube) and cancels out. The total force on each vertical face is the shear stress intensity multiplied by the area over which it operates, and is thus equal to $vx^2$. If we take moments about the edge $D$ we find the force acting along face $AD$ passes through this edge and

having no lever arm it has no moment; the force acting along face $BC$ has a clockwise moment about edge $D$ equal to the product of the force and the lever arm $CD$, or a moment of $vx^3$. The only other forces now operating (longitudinal compression having been ruled out) are those acting along faces $AB$ and $CD$. The force acting along face $CD$ has no moment about edge $D$, and the force acting along face $AB$ must therefore have an anti-clockwise moment about edge $D$ equal to the other moment $vx^3$ which it balances. Operating on a face of area $x^2$ and having a lever arm $x$ the force must be a shear of stress intensity $v$. Similarly by taking moments about edge $A$ or $B$ it can be shown that the shear acting along face $DC$ is also $v$.

It is now clear that the intensity of vertical shear stress at any point in the section is known if the horizontal shear stress at that point has been determined, the two being equal.

We have seen from the discussion on p. 8 that in a beam bent under load all fibres are either compressed or stretched except those lying in the neutral plane. This stretching or compressing is performed by the horizontal shear which itself is caused as we have just seen by the vertical shear.

If horizontal shear could not be resisted each layer of fibres would bend and slide over the next layer as represented diagrammatically in *Fig.* 13. The

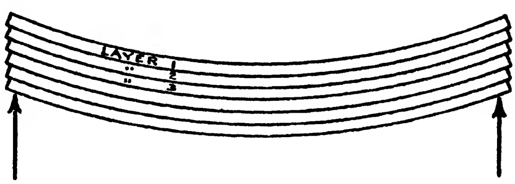



**Fig. 13.**

straining of each layer of fibres by its neighbour is performed through the action of horizontal shear.

Commencing with the extreme fibres, there is obviously no shear on the outside face, and these fibres are compressed to a stress intensity $p$ (*Fig.* 14 (*a*)) by the layer number 2 beneath. The shear between layer 2 and layer 3 how-

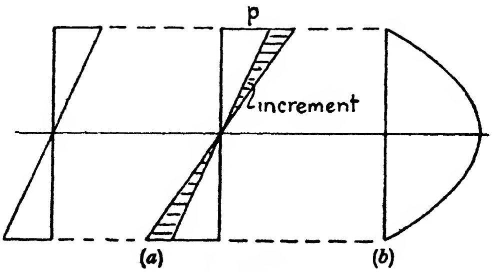



**Fig. 14.**

ever will be greater because not only must it compress layer 2 but it must supply also the shear which layer 2 transfers to layer 1. This shear will thus increase from layer to layer until a maximum is reached at the level of the neutral plane. Thereafter it decreases as the shear is reversed, producing tension instead of com-

pression. On the neutral plane the horizontal shear must be sufficient to stretch all the fibres on one side of itself, and this stretching is balanced by the compression produced on the other side. The horizontal shear between two successive vertical sections must be sufficient to produce the increase in fibre stress from the one section to the next as shown by the cross-hatched area in *Fig.* 14 (*a*). *Fig.* 14 (*b*) shows the horizontal shear stress intensity, the horizontal dimension of the diagram giving the shear at any depth. This dimension, as we may see from the previous discussion, is equal to the area of the cross-hatched portion in *Fig.* 14 (*a*) taken from the level of the point under consideration to the extreme fibre. Thus at the neutral axis where the horizontal dimension (*Fig.* 14 (*b*)) is a maximum the shear stress intensity is equal to the area of the whole of the cross-hatched triangle (*Fig.* 14 (*a*)) taken either above or below this level; and at the extreme fibre the value is zero.

The relationship between *Fig.* 14 (*b*) and the cross-hatched portion of *Fig.* 14 (*a*) is the same as that existing between the B.M.D. and the S.F.D. of *Fig.* 7, it being remembered that the bending moment at any section is equal to the area of the shear force diagram taken on one side of the section in question (see p. 5). In the present case *Fig.* 14 (*b*) is therefore a parabola.

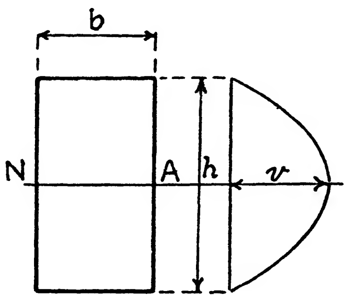



**Fig. 15.**

The horizontal depth of *Fig.* 14 (*b*) gives the horizontal shear stress intensity at any point in the section. The vertical shear stress at the same point will, as we have already seen, have the same value. The total vertical shear on the section must however equal the total external shear $V$. In the case of a rectangular beam of depth $h$ and breadth $b$ as shown in *Fig.* 15, if the maximum shear stress represented by the mid-ordinate of the parabola is $v$ (the diagram now representing vertical shear), the total shear on the section is $\frac{2}{3}v \times ab$ * and this must equal $V$. Therefore in a homogeneous beam

$$v = \frac{3}{2} \times \frac{V}{ab} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or, the maximum shear stress intensity is $1\frac{1}{2}$ times the average.

The following example is purely theoretical, and is given simply to demonstrate a method; the result has no significance. †

* The area of a parabola is $\frac{2}{3}$ height × base.

† As the stress is taken up to the point of rupture, considerably outside the limits of safe working stress, the actual stress is very appreciably different from (less than) the calculated stress. It was stated on p. 8 (Hooke's Law) that strain is proportional to stress within limits; outside of these limits the value for $E$ may vary considerably depending on the material. See also pp. 37 and 38.

EXAMPLE (*Fig.* 16).—A plain concrete bar is supported as a beam and loaded at mid-span in order to test the flexural strength of the concrete. The beam is 2 in. wide and 4 in. deep, the span is 4 ft. and failure occurs by tension in the lower fibres at mid-span under a single load of 216 lb. Find (*a*) the maximum flexural strength of the concrete as calculated from equation (9),† and (*b*) the maximum unit shear stress which was produced in the bar.

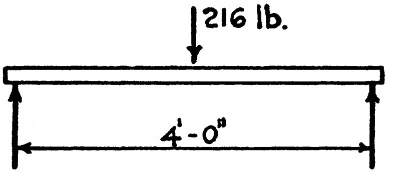



**Fig. 16.**

(*a*) Tensile Stress.

$$I = \frac{2 \times 4^3}{12} = \frac{32}{3} \text{ in.}^4$$

$$y = 2 \text{ in.}$$

$$Z = \frac{32}{3 \times 2} = \frac{16}{3}$$

$$M = \tfrac{1}{4} \times 216 \times 4 = 216 \text{ ft. lb.}$$
$$= 216 \times 12 \text{ in. lb.}$$

$$p = \frac{216 \times 12 \times 3}{16} = 486 \text{ lb. per square inch.}$$

(*b*) Maximum Unit Shear Stress.

$$\text{Total shear, } V = \frac{216}{2} = 108 \text{ lb.}$$

$$\text{Maximum unit shear, } v = \frac{3}{2} \times \frac{108}{4 \times 2} = 20\tfrac{1}{4} \text{ lb. per square inch.}$$

† As the stress is taken up to the point of rupture, considerably outside the limits of safe working stress, the actual stress is very appreciably different from (less than) the calculated stress. It was stated on p. 8 (Hooke's Law) that strain is proportional to stress within limits; outside of these limits the value for $E$ may vary considerably depending on the material. See also pp. 37 and 38.

# CHAPTER II

## CONTINUOUS BEAMS

### Bending Moment and Shear Force Diagrams.

IN Chapter I we dealt with simple systems such as the free-supported beam and the cantilever. Both of these can be solved by the aid of the two equations (5) and (6), the bending moments and shears being determined solely by the external or applied forces, and in both of these cases the bending moment is of one algebraical sign.

We now come to continuous beams in which end fixity or continuity over the supports plays an important part: the bending moment inside any one loaded span will (except in a certain exceptional case referred to later) change in sign, the moment being a positive or sagging one near mid-span and negative or hogging over and near the supports.

The action is not at all difficult to understand if we think first of a number of freely-supported spans, adjacent spans having a common support as shown in the top part of *Fig.* 17; these spans will sag under load so that gaps will open over the supports due to the shortened compression fibres, and the bending moment diagram for each span will be of the nature of those shown in *Fig.* 9, the exact shape depending upon the loading. We now apply bending moments to each of the free ends at all gaps, the magnitude of the applied bending moments being just sufficient to close the gaps. As the bending moments close up the gaps the sag in each beam will be reduced, consequently the curvature and thus the positive or sagging bending moment will be reduced. The bending moment over each support will be a hogging or negative one. Actually, of course, in a continuous beam system the gaps will be prevented from opening by the continuity of the material comprising the beam, but the resulting condition will be exactly the same as if the two sets of moments had been developed independently and superimposed as shown in the lower portions of *Fig.* 17.

In calculating the bending moments we apply this principle of superimposition by determining first the negative moments by the method described below, and then the positive moments which would exist in the freely-supported spans under the same conditions of loading. After making the calculations the simplest method is to draw the " free " bending moments and then measure over each support the value of the negative moment at that section, and join the points obtained by straight lines. This method is shown in the bottom of *Fig.* 17, where the final bending moment diagram is shown shaded. As the beam system has been assumed to have free supports at the two extreme ends the bending moments

at these two supports will be zero. The sections where the bending moment changes from positive to negative (that is, where the B.M. is zero) are termed points of inflexion.

The method of calculating the negative moments at the supports is by writing down from inspection of the loads and spans a series of simultaneous equations which are solved algebraically. This method was published by Bertot in 1855; and since the equations depend upon the relationships existing between the moments at each of three consecutive supports and the span lengths and loadings between these supports, the basis of the method is termed the Theorem of Three Moments.

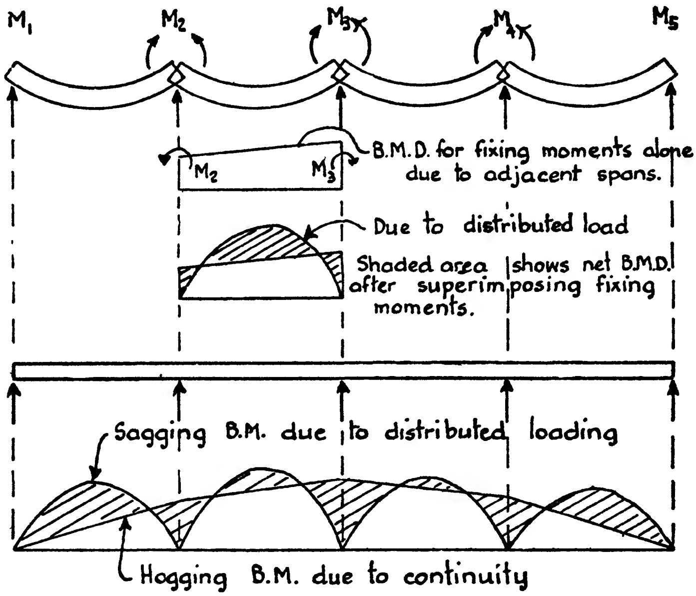



Fig. 17.

The proof of the equations giving the relationships is outside the scope of this book, but it may be found in a number of textbooks on the theory of structures. The equations here given are in several different forms, so as to be convenient for different conditions of loading and support. The proof in its most comprehensive form which makes allowance for variation in beam section within the span (see p. 8), and for the settlement of supports, both of which factors affect the distribution of moments within the system, is available in larger textbooks. Here we must be content to deal with the general case in which the beam section (and thus the moment of inertia) remains constant throughout the system (moderate variations in $I$ may generally be ignored), and where the supports all remain at a constant level: almost all practical cases are considered to fall within this class.

The symbols used are clear from the illustrations. The subscripts to $l$ and $k$ (*Figs.* 18 and 19) refer to the spans.

Beams of Uniform Section ($I$ = constant), having Supports at Constant Level.

Concentrated Loads (*Figs.* 18 and 19).

$$M_2l_2 + 2M_3(l_2 + l_3) + M_4l_3 = -\Sigma W_2l_2^2(k_2 - k_2^3) - \Sigma W_3l_3^2(k_3 - k_3^3) \quad (12)$$

It will be remembered that $\Sigma$ means the sum of all expressions similar to that

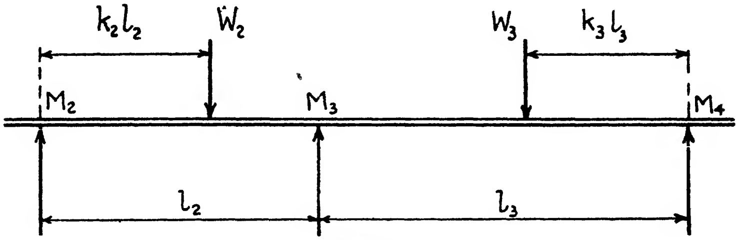



Fig. 18.

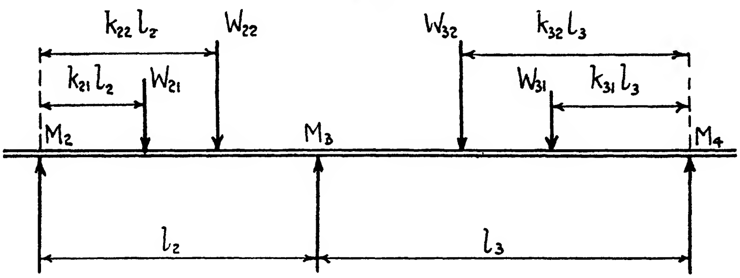



Fig. 19.

following the symbol, and applies to the complete term, so that with the loading shown in *Fig.* 19 the equation becomes

$$M_2l_2 + 2M_3(l_2 + l_3) + M_4l_3 = - W_{21}l_2^2(k_{21} - k_{21}^3) - W_{22}l_2^2(k_{22} - k_{22}^3) \\ - W_{31}l_3^2(k_{31} - k_{31}^3) - W_{32}l_3^2(k_{32} - k_{32}^3). \quad (13)$$

If in any span there is no load the value of $W$ is zero and the term $\Sigma W$, etc., vanishes.

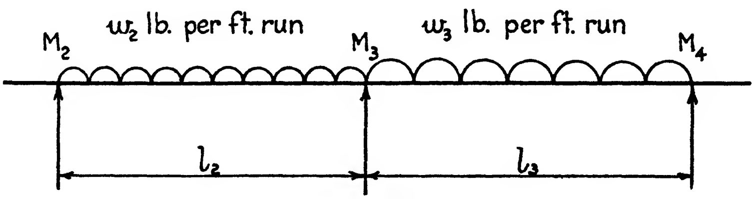



Fig. 20.

Distributed Load, Uniform over any Span (*Fig.* 20).

$$M_2l_2 + 2M_3(l_2 + l_3) + M_4l_3 = - \tfrac{1}{4}w_2l_2^3 - \tfrac{1}{4}w_3l_3^3 \quad (14)$$

If one span is unloaded the value of $w$ in that span is zero and the term involving it vanishes.

If the spans are equal and the uniformly distributed load is constant over both spans the last equation becomes

$$M_2 + 4M_3 + M_4 = -\tfrac{1}{2}wl^2 \quad . \quad . \quad . \quad . \quad (15)$$

Besides solving the case of a beam continuous over several spans the Theorem of Three Moments can be used to determine the fixing moments in the case of a single span with fixed (or " built in ") ends (*Fig.* 21). We assume end fixity to

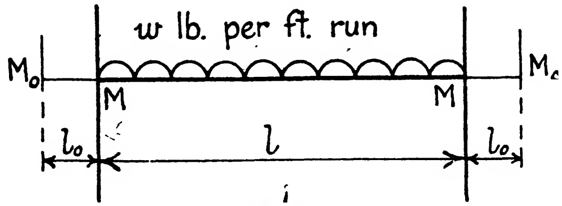



**Fig. 21.**

be caused, or to be replaced, by a very short span at each end so that there are three spans, but the two end spans are each of zero length, and the end moments (as in the case of *Fig.* 17) are each equal to zero. The loading is in the present example uniform, $M_0 = 0$ and $M = M$ by symmetry.*

Then $$M_0l_0 + 2M(l_0 + l) + Ml = -\tfrac{1}{4}wl^3$$
but $l_0 = 0$, so that

$$3Ml = -\tfrac{1}{4}wl^3$$

$$\therefore M = -\frac{wl^2}{12} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The " free " bending moment is a parabola having an altitude of $\frac{wl^2}{8}$. If the fixing moment at each end is $\frac{wl^2}{12}$ the net mid-span moment must be $\frac{wl^2}{24}$.

The same method is also applied where a beam continuous over a number of spans has fixed ends.

In applying the Theorem of Three Moments to any continuous beam system an equation is written down by inspection governing the first three supports, then the equation relating the second, third, and fourth supports, then the third, fourth, and fifth, and so on until the whole system has been covered. There will then be just a sufficient number of simultaneous equations to give a solution.

## Varying Moment of Inertia.

When the beam section varies within the span it affects the distribution of the bending moments. In a continuous beam system the largest bending moments are usually found at the supports rather than at mid-span so that it is economical

* By symmetry. This is a valuable method. In a number of cases mere inspection shows that a system is symmetrical in structure and loading and therefore a number of forces are known to be equal as similar causes and conditions always produce similar effects. (See also p. 67.)

to deepen the section of the beam near the supports by means of haunches as shown in *Fig.* 22. This can generally be done as the maximum headroom is generally required at mid-span—in fact instead of regarding the method as that of deepening the beam at the supports we can fix the beam depth by the requirement there for maximum bending moment and reduce the section at midspan.

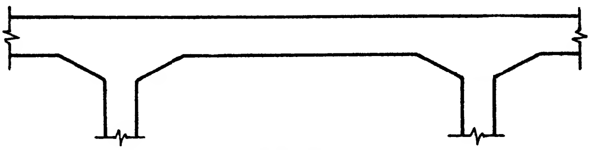


Fig. 22.

The effect of a change in section is to gather to that portion of the beam whose section is relatively increased a larger share of the bending moment, to the relief of the shallower sections. The redistribution of internal stresses is in ordinary cases not great, and unless the change is exceptional the equations already given may reasonably be used. The moments calculated at the deeper sections will be a little under-estimated, and those at the shallower sections will be slightly over-estimated.

### Effect of Settlement of Supports.

A portion of a continuous beam system is shown in *Fig.* 23. If the support $R_3$ is removed two spans are replaced by one, the negative moment at $R_3$ disappears but the positive moment will be much greater than previously existed in either of the smaller spans and the support moments at $R_2$ and $R_4$ will also be greatly increased. If, instead of completely removing $R_3$, the support merely settles a short distance owing to poor foundations, the tendency is in the same direction. Although the negative moment over this support may not com-

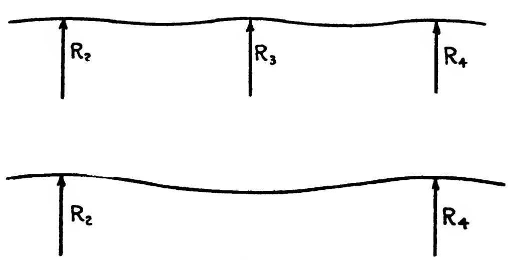



Fig. 23.

pletely vanish it will yet be reduced; the positive moments in the two spans will tend to increase, but the worst effect will be felt in the rapid increase of the other support moments. The limit of this book prevents a discussion of the calculation of the effects of relative settlement, but it is necessary to call the reader's attention to the fact that they exist. It is therefore necessary, in a design where a numbe of consecutive spans are required, to make sure that suitable foundations can be provided if the structure is to be made continuous; otherwise freely-supported

spans should be used as the bending moments are then not affected by reasonable settlements. When a continuous structure has been designed it is then necessary to see that the foundations at the site conform to the conditions assumed in the design.

### Reactions and Shears.

So far we have seen how to obtain equations for the solution of the bending moments occurring over the supports in a continuous beam system. The mid-span moments, as previously mentioned, can be found by drawing the " free " bending moment diagrams and superimposing the support moments on them as shown in *Fig.* 17. In order to design the beam we must also be able to determine the shears. These will be somewhat different from those in a freely-supported span under the same load, although the two end shears in any one span will always together (taken algebraically) be equal to the load on that span. The maximum shears in the span will always occur at the supports.

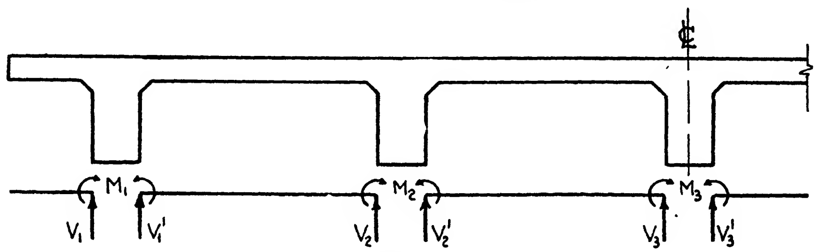



**Fig. 24.**

*Fig.* 24 represents the section of a part of a deck girder system. Uniform section and load are assumed in the working below.

$M_1$ and $M_5$ are equal and are known as being simple cantilever moments.

$M_2 = M_4$ by symmetry.

$V_1$ is the simple cantilever shear and is therefore known.

$$M_1 + 4M_2 + M_3 = -\tfrac{1}{2}wl^2$$
$$M_2 + 4M_3 + M_4 = -\tfrac{1}{2}wl^2.$$

As $M_2$ and $M_4$ are equal these and $M_3$ are easily determined. The moments should be indicated by arrows, and in writing down the equations for shears the moment signs should be decided by the directions of the arrows (clockwise or anti-clockwise) and not according to whether the moment is hogging or sagging. The individual span has now become the unit.

To determine $V_1'$ take moments about support No. 2, when because of equilibrium

$$M_1 - M_2 - V_1'l + \frac{wl^2}{2} = 0 \quad . \quad . \quad . \quad . \quad (17)$$

This will give $V_1'$.

Should any concentrated loads occur on the span, the moments of these about support No. 2 would be included in the equation.

To determine $V_2$

$$V_1' + V_2 = wl + \text{any concentrated loads}$$
$$V_2 = wl + \text{,,} \qquad \text{,,} \qquad \text{,,} \quad - V_1' \quad . \quad . \quad (18)$$

A general equation for determining the reactions and shears when the bending moments are known will now be given (see *Fig.* 25).

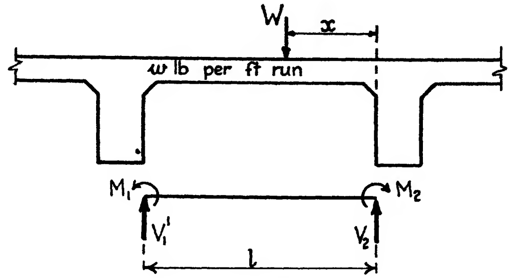



**Fig. 25.**

If clockwise moments are taken as positive,

$$M_2 - M_1 + V_1'l - \frac{wl^2}{2} - Wx = 0 \quad . \quad . \quad . \quad (19)$$

This equation gives $V_1'$.

Resolve vertically, giving

$$V_1' + V_2 - wl - W = 0 \quad . \quad . \quad . \quad . \quad (20)$$

This equation gives $V_2$.

With the loading known and the shears at the supports determined, the shear force diagram can be drawn in the same way as for a freely-supported beam.

The reaction at any support is of course equal to the sum of the shears immediately on either side together with any load directly over it.

**Example of the Use of the Theorem of Three Moments.** *Fig.* 26 represents a continuous floor beam, the two 20-ft. spans supporting the floors of two rooms, the 10-ft. span the corridor, and the 3-ft. cantilever a small balcony. The loads are those due solely to the weight of the flooring supported by the beam and

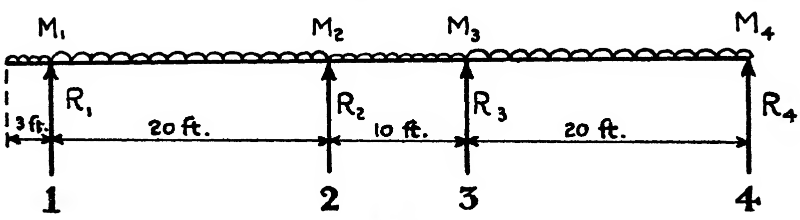



**Fig. 26.**

the beam's own weight. These are 100 lb. per foot run on the middle span and the cantilever, and 200 lb. per foot run on the two longer spans. Draw the dead load * bending moment and shear force diagrams. The floor beam is of constant section (constant $I$).

Taking hogging moments as negative,

$$M_1 = -300 \times 1\tfrac{1}{2} = -450 \text{ ft. lb.}; \quad M_4 = 0;$$

$$l_{12} = 20 \text{ ft.} = l_{34}; \quad l_{23} = 10 \text{ ft.}$$

The suitable equation to use is (14) from p. 17.

* Dead Load—see p. 24.

Considering supports 1, 2 and 3,

$$M_1 l_{12} + 2M_2(l_{12} + l_{23}) + M_3 l_{23} = -\tfrac{1}{4} \times w_{12} \times l_{12}^3 - \tfrac{1}{4} \times w_{23} l_{23}^3 \quad (14a)$$

$$20M_1 + 60M_2 + 10M_3 = -\tfrac{1}{4} \times 200 \times 20^3 - \tfrac{1}{4} \times 100 \times 10^3$$

$$\therefore -9{,}000 + 60M_2 + 10M_3 = -400{,}000 - 25{,}000.$$

Divide through by 10,

then
$$6M_2 + M_3 = -41{,}600 \text{ ft. lb.} \quad (21)$$

Considering supports 2, 3, and 4,

$$M_2 l_{23} + 2M_3(l_{23} + l_{34}) + M_4 l_{34} = -\tfrac{1}{4} w_{23} l_{23}^3 - \tfrac{1}{4} \times w_{34} \times l_{31}^3 \quad (14b)$$

$$10M_2 + 60M_3 + 0 = -25{,}000 - 400{,}000.$$

Divide through by 10,

then
$$M_2 + 6M_3 = -42{,}500 \text{ ft. lb.} \quad (22)$$

Multiply equation (21) by 6, then

$$36M_2 + 6M_3 = -249{,}600$$

Subtracting
$$-35M_2 = +207{,}100$$

$$\therefore M_2 = -5{,}920 \text{ ft. lb.}$$

Substituting this value for $M_2$ in (22) we get,

$$6M_3 = -42{,}500 + 5{,}920$$

$$\therefore M_3 = -\frac{36{,}580}{6}$$

$$M_3 = -6{,}096 \text{ ft. lb.}$$

The maximum ordinates to the "free" span bending moments would be:

for the 20-ft. span, $\frac{1}{8} \times 200 \times 20^2 = 10{,}000$ ft. lb.

for the 10-ft. span, $\frac{1}{8} \times 100 \times 10^2 = 1{,}250$ ft. lb.

For uniform load the free-span bending moments would be parabolas. These have been drawn in *Fig.* 27, the support moments have been scaled over the supports, and joined up giving the net bending moments at all sections.

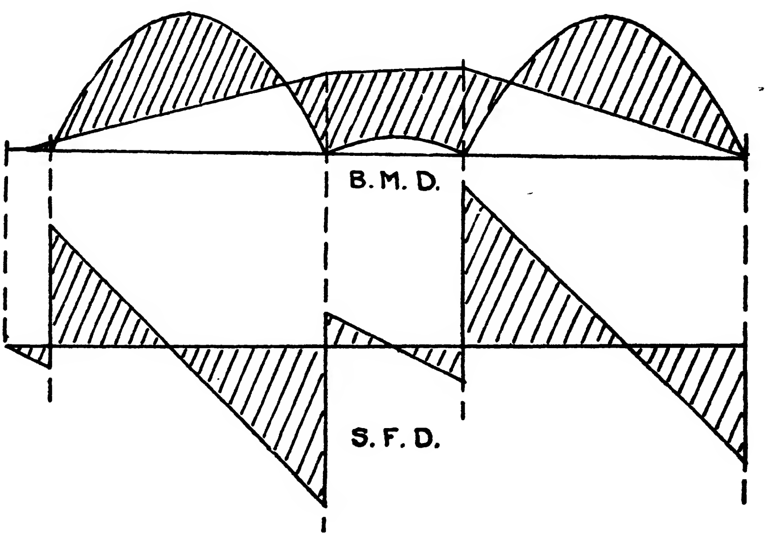



**Fig. 27.**

The spans and loads in this example have been selected in such a way as to bring out the exceptional case referred to on p. 15. The negative bending moments over supports 2 and 3 are of such magnitude as completely to swamp the "free" span bending moment in span 2–3, with the result that negative moment occurs throughout the whole of the small span. This is due to the relative span lengths, and in a minor degree also to the loading: wherever a short span is flanked by appreciably longer spans the same condition may reasonably be expected.

### Shears and Reactions.

In following this work the student will notice the importance of using a sound system of notation. Using a notation similar to that in *Fig.* 24, $V_1 = 300$ lb.

Take moments about support No. 2 (moment signs according to arrows in *Fig.* 24).

$$M_1 - M_2 - V_1'l_{12} + \frac{w_{12}l_{12}^2}{2} = 0 \qquad \text{(17a)}$$

$$450 - 5{,}920 - 20V_1' + \frac{200 \times 20^2}{2} = 0$$

$$\therefore\ V_1' = \frac{450 + 40{,}000 - 5{,}920}{20} = 1{,}727 \text{ lb.}$$

$$V_2 = w_{12}l_{12} - V_1' \qquad \text{(18a)}$$

$$= 200 \times 20 - 1{,}727 \qquad = 2{,}273 \text{ lb.}$$

The reaction at support No. $1 = V_1 + V_1'$

$$R_1 = 300 + 1{,}727 = 2{,}027 \text{ lb.}$$

Take moments about support No. 3.

$$M_2 - M_3 - V_2'l_{23} + \frac{w_{23}l_{23}^2}{2} = 0 \qquad \text{(17b)}$$

$$5{,}920 - 6{,}096 - 10V_2' + \frac{100 \times 10^2}{2} = 0$$

$$\therefore\ V_2' = \frac{5{,}920 - 6{,}096 + 5{,}000}{10} = 482 \text{ lb.}$$

$$V_3 = w_{23}l_{23} - V_2' \qquad \text{(18b)}$$

$$= 100 \times 10 - 482 \qquad = 518 \text{ lb.}$$

The reaction at support No. $2 = V_2 + V_2'$

$$R_2 = 2{,}273 + 482 = 2{,}755 \text{ lb.}$$

Take moments about support No. 4.

$$M_3 - M_4 - V_3'l_{34} + \frac{w_{34}l_{34}}{2} = 0 \qquad \text{(17c)}$$

$$6{,}096 - 0 - 20V_3' + \frac{200 \times 20^2}{2} = 0$$

$$V_3' = \frac{6{,}096 + 40{,}000}{20} = 2{,}305 \text{ lb.}$$

$$V_4 = w_{34}l_{34} - V_3' \qquad \text{(18c)}$$

$$= 200 \times 20 - 2{,}305 = 1{,}695 \text{ lb.}$$

The reaction at support No. $3 = V_3 + V_3'$

$$R_3 = 518 + 2{,}305 = 2{,}823 \text{ lb.}$$

The reaction at support No. $4 = V_4$

$$R_4 = 1{,}695 \text{ lb.}$$

The student is advised to consider the effects of the span proportions on the moments. A careful study of the results calculated will yield useful information: the shears should be compared with those which would have occurred had the spans all been freely supported. But for the cantilever at one end the system would have been symmetrical, therefore the effect of the cantilever on the moments and shears can be studied. The student should then work several examples himself. Since this will be an exercise in the method of applying the Theorem of Three Moments rather than one in arithmetic, round figures for loads and span lengths should preferably be employed. Care must be taken with the algebraic signs. Sketches will help.

## Influence Lines.

If the condition of loading on a continuous beam system were definitely fixed so that no variations could take place, the Theorem of Three Moments could be applied once to the system and the results would be final. We have in a number of cases, indeed in the majority, to consider variable conditions of load. There is first of all the load of the structure itself together with whatever additions contribute to a permanent and unchanging state of stress. These loads are termed " dead " loads; there is no movement in them. Other loads which will be variable such as those due to wind, snow, traffic, merchandise—any superimposed loads which in any way move or change are termed " live " loads.*

For the condition of dead load one solution of the system by the Theorem of Three Moments will suffice. The live loads however require a new solution for every changed condition or combination of loads, and since the number of these may be very great something must be done to limit the amount of work which would be necessary for a complete set of solutions. Since we are concerned only with the maximum stresses, those for which our sections must be designed, we may limit the number of solutions necessary to those giving the maximum stresses—tension and compression of fibres due to flexure, and shear —in the critical sections. In a freely-supported beam the critical sections would under normal loadings be, for fibre-stress mid-span, and for shear the supports. In a continuous beam system the problem is a little more complicated as a load placed at any point in the system will affect all the other sections in varying ways and by varying amounts. For maximum positive moment in any span under uniform live load the span itself should be loaded, together with alternate spans throughout the system (*Fig.* 28)—the immediately neighbouring spans should be unloaded; the maximum positive moment will then occur at mid-span (section *XX*). For maximum negative moment, which will occur at a

* When live loads are applied very suddenly the stresses immediately set up are in excess of those arising from a gentle application of the load: for this reason with certain types of load, which may also introduce vibration, we introduce an impact factor to increase the " static " load.

support, two adjacent spans should be loaded, and thereafter alternate spans on either side (section $X$ in *Fig.* 29). In both cases the remoter spans have little effect and their influence is often neglected. Where moving concentrated loads

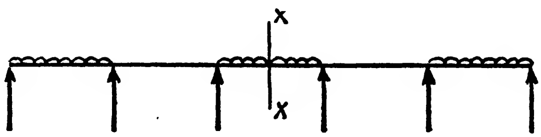



Fig. 28.

have to be considered their influence must be known for different positions in the system.

The difficulty of all these complications is met by the construction of what are termed " Influence Lines." An influence line is a line drawn for any one

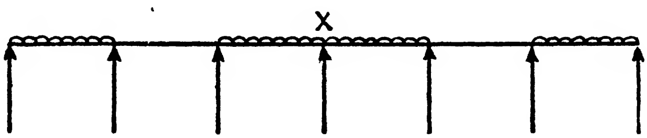



Fig. 29.

section in the system to show the effect on that section of a unit load placed successively at all other points in the system. By effect is meant the bending moment, shear, or fibre stress, whichever function the influence line is constructed to exhibit ; thus an influence line for bending moment at section $XX$ in the two-span system in *Fig.* 30 shows by the vertical height of the curve above the

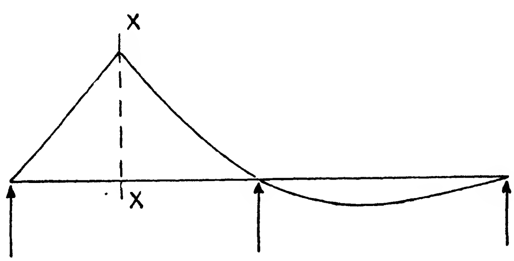
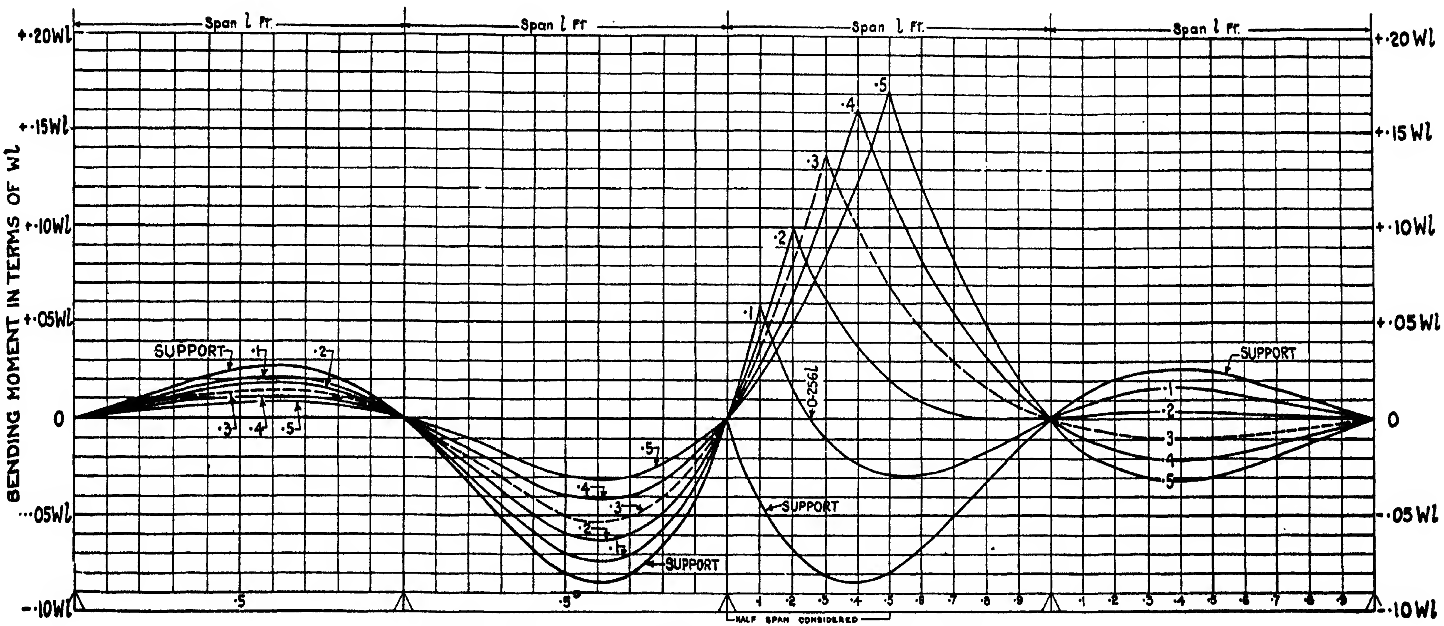



Fig. 30.

base line at any point the effect of a unit load placed at that point, on section $XX$. Influence lines are composed of straight lines or smooth continuous curves, and the student is advised as an exercise to sketch out a number of curves for himself. A unit load can be placed successively at three or four sections in each span and the bending moment ascertained at the section for which the influence line is drawn by superimposing the support moments found by equation (12) on the " free " moments in the manner described on p. 15.

An influence line drawn for any section will show at once which portions




BENDING MOMENT INFLUENCE LINES FOR MOVING POINT LOAD

EACH CURVE IS FOR THE SECTION OF BEAM BELOW ITS CUSP.

Fig. 31.—Intermediate Span of a Long Series of Equal Spans.

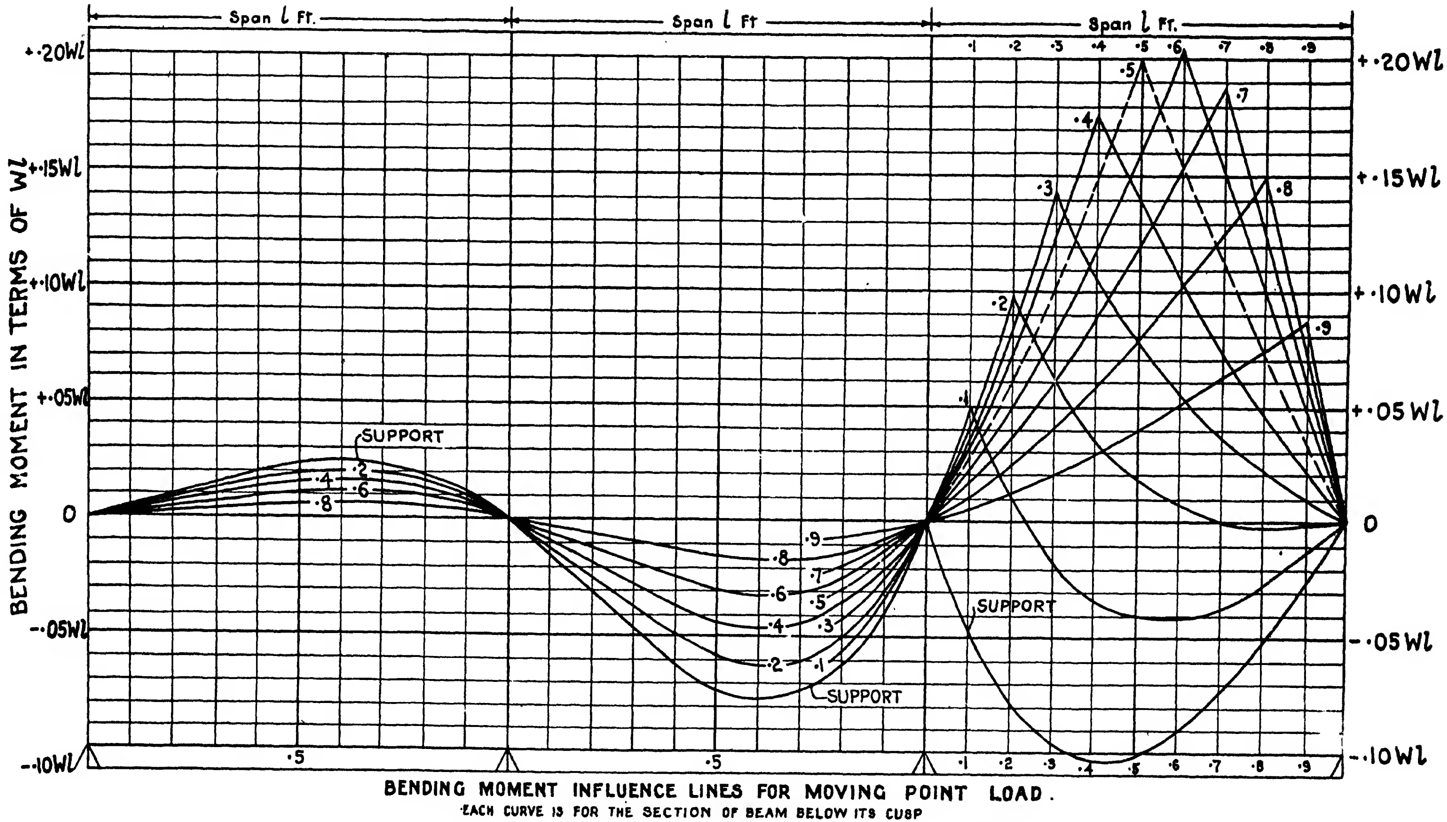



Fig. 31A.—End Span of a Long Series of Equal Spans.

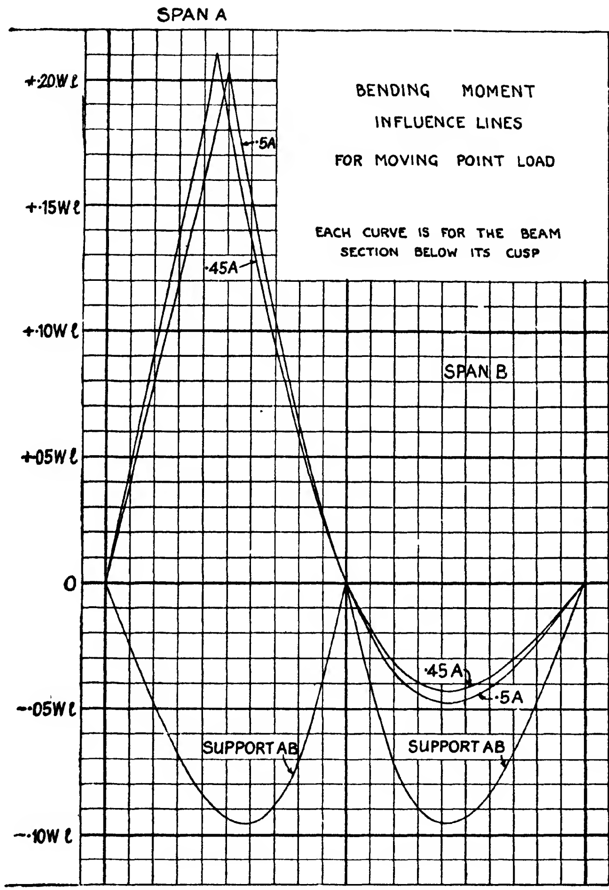



Fig. 31B.—Two Equal Spans.

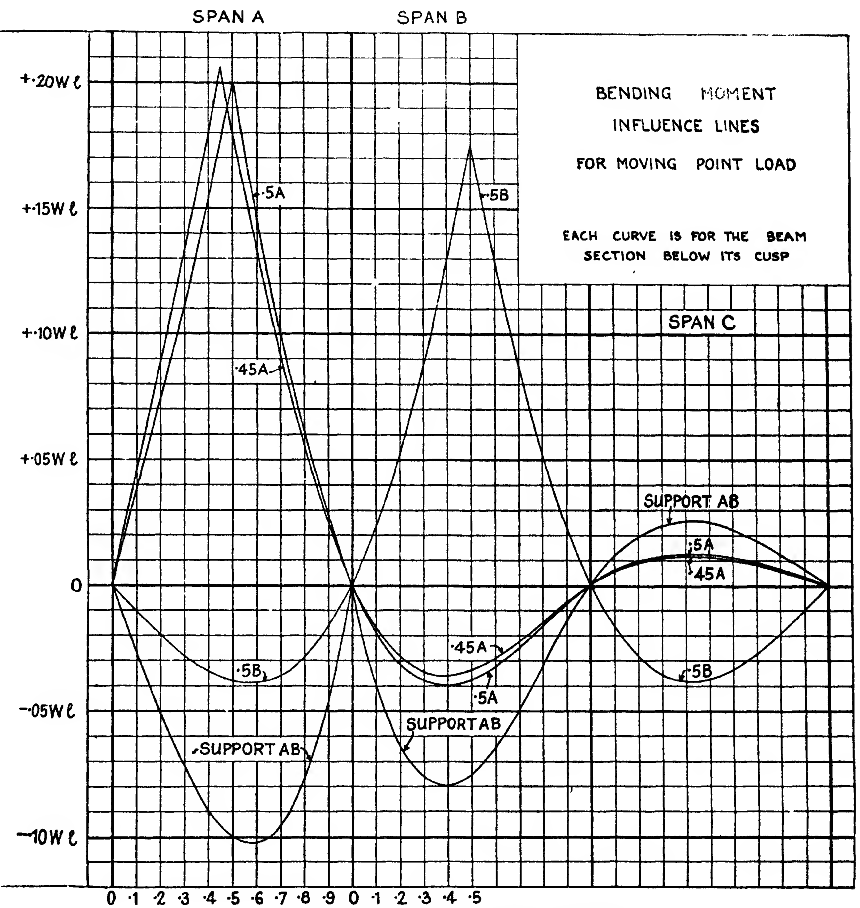



Fig. 31C.—Three Equal Spans.

of the system must be loaded to produce the most severe condition ; and it will show upon examination exactly the magnitude of the effect the loads will have upon that section. Influence lines can be drawn for any system : it is unnecessary to draw them for freely-supported spans, and in continuous beams it is necessary to draw them only for the critical sections. Standard influence lines for a large number of systems are given in Griot's Tables,* and influence lines for varying numbers of equal spans are given in several well-known textbooks. Four only are given in this book : *Fig.* 31 gives influence lines for bending moment at various sections in an " infinite " series of equal spans of constant $I$ (in general this may be used for any system of more than 3 equal spans), and *Fig.* 31A gives influence lines for bending moment at various sections in the end span of a series. *Fig.* 31B is for two, and *Fig.* 31C for three equal spans.

It will be noticed that a cusp occurs in each curve over the section for which the curve is drawn : the ordinate at this point shows, of course, the effect on the section of a load placed just there. Where the possible live load consists of a uniformly distributed load plus a single concentration, the uniform load would be placed over all portions of the system where the effect (moment or shear as the case may be) is of one sign (positive or negative), and the concentrated load would be placed at the point of maximum ordinate—in the case of positive moment, immediately below the cusp.

An example of the use of influence lines, using *Fig.* 31, is given in Chapter VIII (see p. 86).

The maximum live load shear at any given section occurs when the span carries a distributed load to one side only of the section, for a distance extending from the section to the farther support, and any moving concentration is placed at the section itself. This applies to simply-supported and also continuous beams.

* " Kontinuierliche Träger Tabellen," 4 Auflage. (Gustav Griot, Zürich.)

# CHAPTER III

## CONCRETE

CONCRETE as we are to understand it is made with cement, suitable aggregate such as clean sand and gravel or broken stone, and water, all mixed together in suitable proportions. Let us consider these in order.

### Cement.

Cement is the adhesive agent which binds together the sand and stone or other aggregates. Dry cement is an inert powder, but on the addition of water a chemical action takes place which causes the paste thus formed to set. The object of the concrete maker is to coat every surface of every particle of aggregate with a film of this paste, so that each particle will adhere to those in contact with it. Any cement not properly distributed throughout a concrete mix will form isolated globules of neat cement ; it will not then be performing its function of binding together particles of aggregate and is thus wasted so far as the strength of the concrete is concerned. Although the quantity of water required during mixing must not be excessive, during the hardening process water is beneficial —indeed the hardening can take place only in the presence of moisture, so that the concrete should not be allowed to dry out quickly (see p. 34).

There are several kinds of cement. The commonest and best known is ordinary Portland cement which is a finely-ground powder made by burning a mixture consisting principally of clay and chalk, or other materials containing silicates and aluminates of lime. Concrete employing it takes its initial set in something over half an hour and the final set is required by the latest edition of the British Standard Specification to take place in less than ten hours. In warm weather the final set generally occurs in about three or four hours. During 24 hours or so after the final set the rate of hardening is rapid, the action being hastened in warm weather and much delayed with low temperatures. Most of the hardening takes place in the first month, but the concrete continues to harden slowly for a year or more.

Rapid-hardening Portland cement meets the need for a cement of high quality which provides an early high strength concrete. In normal temperatures its 7-day strength is greater than the 28-day strength of concretes made with ordinary Portland cement. The ultimate strengths are much the same. The early strength of rapid-hardening Portland cement concrete is somewhat delayed in very cold weather. Most rapid-hardening Portland cements acquire their special properties from the extra fineness of their grinding, and this accounts for their slightly increased cost over that of ordinary Portland cement.

One other cement is important enough to be included in this brief survey. Aluminous cement, which is much more expensive, provides a concrete with a very high early strength : it can be put into use 24 hours after it is placed. The chemical action in setting generates quickly large quantities of heat, and moisture from the concrete tends to evaporate rapidly. Since the presence of moisture is requisite to the proper hardening of the concrete it is necessary for the concrete to be kept saturated from the time it sets until the following day. Aluminous cement concrete can be usefully employed where the time available for construction is very short—such as in the lengthening of piles and in the construction of sea and river structures between the intervals of tides. A valuable property of concrete made with aluminous cement is its resistance to the action of sea water and to attack from some of the solutions which are injurious to ordinary concretes.

All cements should be protected from damp as this destroys the essential properties and in a short time renders the cement useless.

## Aggregates.

The next ingredient is the aggregate, which should be composed of hard, clean grains of an inert material. In order to give a solid concrete the aggregate should be well graded—that is, it should be composed of grains of varying sizes which will pack well together ; round shapes are eminently suitable, but flat or flaky particles are bad. Although a large number of sizes are necessary and the best concrete would be obtained by the use of a large number of sieves for grading, in practice we have to be content with two classes, fine and coarse, with the proviso however that the materials must be well graded throughout the range of sizes specified within each class. Fine aggregate generally consists of sand, the grains ranging from $\frac{3}{16}$ in. down, but very fine particles (termed flour) which would be capable of displacing the cement should be carefully excluded. Coarse aggregate ranges from $\frac{3}{16}$ in. to $\frac{3}{4}$ in. for ordinary reinforced concrete work, but the size may be taken up to 2 in. or even more for mass work. Gravel, broken stone, or crushed rock provide suitable aggregates.

It is essential that the aggregates be thoroughly clean, free from dust and any organic impurities. Many concrete troubles have been traced to the use of unsuitable aggregates.

The water must be fresh and clean, and in the mix should be used sparingly. Further reference to the quantity will be made later.

## Proportions.

The object in selecting the proportions is to obtain a workable mix as dense as possible so that there are no voids—no " honeycombing " as we say. To be workable the materials should not be too angular or " harsh," and the concrete should not be deficient in mortar. This latter fault is sometimes brought about through the bulking effect of the sand, which should be allowed for. When sand is dry it will, if shaken down, pack close : when moist it " bulks " or increases in volume, the various particles riding up on each other by friction. When in a saturated condition the friction is reduced by lubrication of the grains and the sand is restored to the condition of minimum volume. In practice

all sands contain some moisture and there is therefore some bulking when the sand is being gauged. In the process of mixing the sand is lubricated by the cement and water and shrinks in bulk. If, therefore, the sand as gauged is just sufficient or little more than sufficient to fill the voids in the coarse aggregate, the resulting concrete is likely to be deficient in mortar and therefore porous. An average coarse aggregate may have about 43 per cent. of voids, and it is usual to specify 50 per cent. of sand, which allows a small surplus for the fact that an exactly ideal distribution can never be attained. Owing to the bulking effect, which may sometimes be appreciable, it may sometimes be necessary to add more than 50 per cent. by volume of sand. The cement fills the voids and coats the sand grains, but adds little or nothing to the volume, and the mortar so formed adds very little to the volume of the coarse aggregate; a 1 : 2 : 4 (volume of cement : sand : gravel) mix therefore provides only slightly more than 4 units by volume of concrete.

The cement bulks in its dry condition and its volume varies greatly with the way in which it is handled and gauged; it is therefore better always to gauge cement by weight rather than by volume, as far as possible using a whole number of standard bags to a batch—in this way 90 lb. of cement is assumed to be equivalent to a cubic foot, and the term 1 : 2 : 4 mix would mean a mix having proportions in the ratio of 90 lb. (cement) : 2 cb. ft. (sand) : 4 cb. ft. (gravel). This is a very common mix for structural work. For watertight concrete for tanks, etc., a 1 : 1½ : 3 mix is generally used, while for mass concrete such as employed in large foundations a 1 : 3 : 6 mix is common. Sometimes to avoid the dangers of bulking mixes such as 1 : 2½ : 4 and 1 : 3 : 5 are used. In important work it is wise to experiment in the laboratory with the materials to be employed and to vary the proportions of fine to coarse aggregate until the densest possible mix is obtained. The special requirements of the concrete, such as stress or watertightness, generally determine the proportion of cement to sand, and when this has been settled changes in grading should be made by changing only the coarse aggregate content.

### Water.

Excess water giving a sloppy concrete leads to loss of strength and in some cases to honeycombing. Sufficient water must be provided to give a thoroughly workable mix, and this is specially important where the concrete has to be worked around reinforcing bars; but as soon as the mix has become workable no more water should be added. The strength of concrete, at least in the early months, depends largely on the water-cement ratio, and if through a change in aggregate or conditions of placing it becomes necessary to increase the water content, and it is necessary for the strength to remain constant, the cement should be increased in equal proportion by volume. *Fig.* 32 shows a curve relating strength to water-cement ratio, as determined first at the Structural Materials Research Laboratory, Chicago, and verified at many other research stations. The maximum strength is obtained with a very dry mix which is unworkable, and a small amount of strength must be sacrificed for other qualities equally important. The exact shape of the curve, and the exact quantity of water required, vary with the aggregates and with the mix. Lean mixes require more water than rich

ones to make them workable. An average 1 : 2 : 4 concrete requires between five and six Imperial gallons of water to 90 lb. of cement.

## Mixing and Placing.

Concrete should be thoroughly mixed, preferably in a machine and for at least 2 minutes, and should then be placed in final position immediately, before it has time to segregate or develop its initial set. The wet concrete should be well worked into position so that it packs densely without imprisoning air. Scum is likely to form on the top, especially if there is the least amount of excess

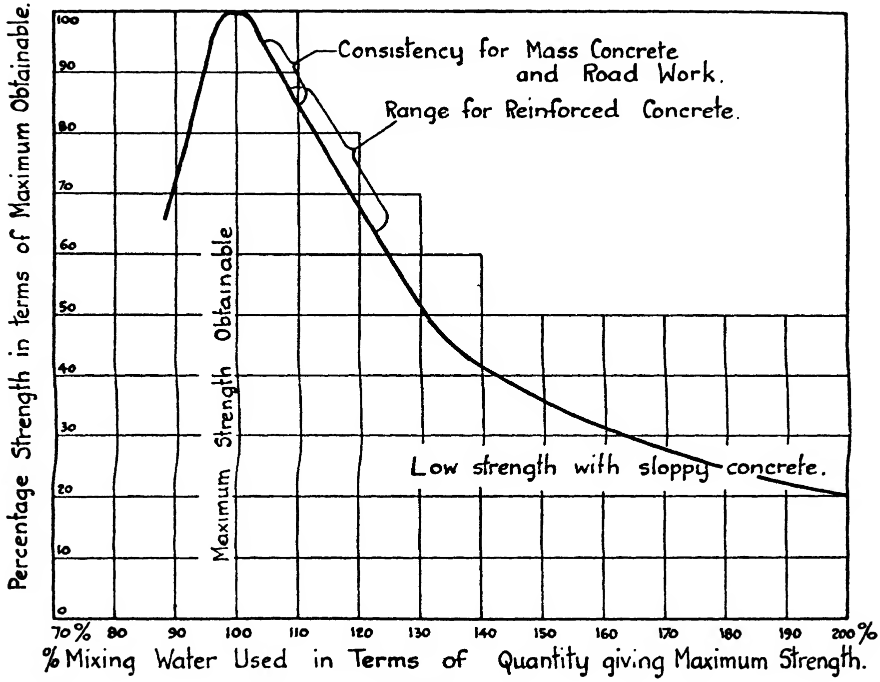



Fig. 32.

water ; this scum is formed of cement particles, and their presence at the top not only indicates a weakening of the body of concrete below but also constitutes a plane of weakness if more concrete is later to be bonded at this section. For this reason it is necessary when making a joint like this to hack away the old surface, which should then be cleaned and wetted and the joint made with a rich mortar.

In important structures 6-in. test cubes are prepared from concrete taken from between the forms at the time of placing ; these are then stored under known conditions and generally tested by crushing at the ages of 7 and 28 days, although sometimes tests are made at 3 and 7 days. This forms a check on the strength of the concrete in the structure itself. Occasionally test cylinders

are used instead, having a height equal to twice the diameter: the crushing strength of such cylinders is about three-quarters of the cube strength.

### Curing.

The hardening process commences from the time of set and continues more or less indefinitely, although after the first month or so the rate of increase is gradual and becomes asymptotic. The hardening is dependent upon a crystallising action which uses some of the moisture in the concrete. Owing to evaporation and absorption by forms or other bodies in contact with the concrete it becomes necessary to replenish the moisture which has been lost or the hardening process will be hindered. Sun and wind promote evaporation, and the concrete should be protected by forms, wet canvas, wet sand, or in some other way, for as long as possible from their action.

During hardening the concrete also undergoes a volumetric change. If immersed in water slight swelling may occur, but under normal conditions the concrete shrinks. This is due (1) to the cement, and (2) to the wetness of the original mix. Other things being equal a rich mix shrinks more than a lean one, and a wet mix * much more than a dry one. Where concrete in a large unit is restrained cracking will occur, and where construction joints are provided these tend slightly to open. The linear contraction of concrete due to shrinkage is generally from 0·0003 to 0·0004 of the length, about 0·0001 occurring in the first month.

The total amount of shrinkage is much the same with all cements, but the rate of shrinkage depends partly on the rate of hardening—early hardening is generally accompanied by early shrinkage. The prevention of quick drying out by proper curing reduces the shrinkage.

Since reinforcing steel resists shrinkage a heavily reinforced section is less liable to shrinkage than one of plain concrete; moreover, because of the bond between concrete and steel, the shrinkage cracks become well distributed and may be kept invisible—in an unreinforced strip of concrete large shrinkage cracks occur at fairly regular intervals. With both mass concrete and reinforced concrete joints should be provided where the shrinkage can be taken up. These joints allow also for expansion and contraction under temperature changes. The coefficient of linear expansion of concrete is about 0·000006 per deg. F.

### Concrete under Stress.

Concrete is capable of resisting compression but is weak in tension, and it is for this reason that reinforcement is provided where tensile stresses will occur. Like all other materials concrete is elastic and the modulus of elasticity, $E$, varies for average concretes from about 2,000,000 to about 5,000,000 lb. per square inch. If the concrete is under stress for only short periods of time strain is proportional to stress and recovery after the load is removed is almost complete. Under sustained load, however, the strain increases without increase of stress and this additional amount of strain remains after the stress is removed. This

* Note that the wetness here applies to the water content at the time of mixing, and not to water applied subsequently, which is beneficial.

residual strain is termed "creep" or "plastic yield," and the amount may be much greater than the elastic strain; for this reason the value of $E$ used in design is taken lower than the instantaneous value; for normal mixes it is taken as 2,000,000, and a little more for the very rich mixes. The plastic yield is roughly proportional to stress, and concrete is much more susceptible to this deformation in the early stages of hardening; about three-quarters of the final amount of yield will occur, under load, in the first year. Apart from the use of a reduced value for $E$, no special account is taken of plastic yield in design. The strength of concrete will be discussed later.

## Reinforcing Steel.

Steel bars are provided for the purpose of taking tensile stresses. Plain round bars are generally selected, and they should be clean and free from mill scale, rust, grease, and paint. In this condition the concrete adheres to the bar and the grip is accentuated by the shrinking on of the concrete. "Bond" is the term applied to this joint action, and good bond is an essential feature in a reinforced concrete member. The coefficient of linear expansion of steel is about 0·0000065 per deg. F., which is little different from that of concrete (0·000006), and they therefore act together without much stress being set up by tendencies to relative deformation owing to changing temperature.

It is important that reinforcing steel should be placed accurately and well secured in position by soft iron wire (No. 16 S.W.G. is suitable), and care should be taken that the reinforcement is not displaced during the depositing and working of the concrete. In detailing the reinforcement plenty of space should be left between bars for each to be well surrounded with concrete. The minimum clear space in the most concentrated groups of bars, such as occur in heavily reinforced beams, should be greater than the maximum dimension of the coarse aggregate. The minimum clear space is generally taken as 1 in., or the size of the bar, whichever is greater. A sound practice adopted by many designers is to provide a clear space of 1½ bar diameters (see p. 50). Ample space should also be left between the bar and the forms, or in the case of top steel in a slab between the bar and the surface. The clear concrete cover to all bars should be at least ¾ in. for slabs, 1 in. for columns, walls and beams, and at least 2 in. for sea work.

From the designer's viewpoint the primary property of reinforcement is the yield-point stress upon which the working tensile stress is based. The latter should not exceed half the yield-point stress and in no case should it be greater than 27,000 lb. per square inch in buildings and much less in exposed structures or those containing liquids. British Standard 785 (1938) does not specify a yield-point stress for either mild steel or cold drawn wire, but in practice maximum working stresses of 18,000 lb. per square inch and 27,000 lb. per square inch respectively are usual. For hot-rolled bars other than mild steel, B.S. 785 requires the minimum yield-point stress, depending on the size of the bar, to be: for medium-tensile steel, 39,200 lb. per square inch to 43,680 lb. per square inch; for high-tensile steel, 47,040 lb. per square inch to 51,520 lb. per square inch. British Standard 1144 (1943) requires a minimum yield-point stress for twin-twisted bars (all sizes) of 54,000 lb. per square inch; and for twisted square bars 60,000 lb. per square inch for bars of ⅜ in. and above, and 70,000 lb. per square inch below this size.

## Formwork.

The requirements of good formwork are that it should provide a firm and regular surface which will retain the concrete, that there should be no gaps through which the grout can leak, that it should resist distortion under the weight and pressure of the wet concrete, and that it should not absorb moisture to any appreciable extent from the concrete or stick to the surface. Timber or steel is generally used. It should be constructed so as to facilitate stripping without damage to either the formwork or the concrete. To prevent adhesion between the forms and concrete the inside faces of the forms are usually treated with oil or grease. As a cheap alternative timber forms are saturated on the inside with water or treated with limewash. Steel shuttering provides a good smooth surface if the forms are well aligned. There is the disadvantage in cold weather that they are less capable of conserving the heat generated during hardening, and therefore the hardening process may be retarded. Although timber forms are frequently re-used, steel forms have a much longer life, but are less adaptable and the initial cost is greater.

During the erection of formwork allowance should be made for slight settlement of the supporting falsework and compacting in the joints as the load from the wet concrete is imposed.

Falsework should not be struck until the concrete has hardened sufficiently to carry safely its own dead load. Those forms which are not required to support any load may be stripped at an earlier stage. It is difficult to give periods which should elapse before stripping, as much depends on the temperature, the nature of the structure, and the quality of the cement. The following times should only be taken as a rough guide for ordinary Portland cement and normal temperature; the decision should be left to an experienced engineer: (*a*) beam sides, walls, columns, 3 to 7 days; (*b*) floor slabs, 7 to 10 days; (*c*) under-sides of beams, 2 weeks or more for long spans.

With rapid-hardening Portland cement and normal temperature (*a*) might be reduced to 1 day, (*b*) to 5 days, and (*c*) to 5 to 8 days. In cold weather these times would be extended. Stripping renders concrete more susceptible to drying out and thus operates to a certain extent against the beneficial effects of curing unless these can be provided in some other way.

If the formwork has been properly prepared and erected not much work on the exposed concrete face will be necessary. Where a smooth, plain face is required one or two small holes may need stopping with rich mortar rubbed in with a wood float; board marks may be rubbed off with a wood block while the concrete is green, or later with blocks of carborundum. It is a mistake to wash the surface with cement as this skin tends to flake and craze.

A suitable architectural feature can be made of the board marks if these are regular, and they may be suitably emphasised by using for forms boards with chamfered edges.

When forms are intended to be re-used they should be thoroughly cleaned immediately upon stripping.

# CHAPTER IV

## REINFORCED CONCRETE UNDER STRESS

For practical purposes, within the range of ordinary stresses, and except for the effect of plastic yield, strain is proportional to stress. If the modulus of elasticity is known and a bar specimen of a material is loaded in tension or compression and the strain is measured, the stress can be deduced; if the load (and therefore the stress) is known and the strain is measured, the modulus of elasticity ($E$) may be deduced.

As explained on p. 35 the value of $E$ for the most ordinary concrete mixes (termed $E_c$) is generally taken at 2,000,000 lb. per square inch, which figure includes some allowance for the effect of plastic yield: with the richer mixes higher values, such as 2,500,000 and 3,000,000, are generally used. The value of $E$ for mild steel reinforcing bars (termed $E_s$) may be taken as 30,000,000. Thus if equal lengths of concrete and steel are each stretched or compressed by equal amounts the stress in the steel will be $\frac{30,000,000}{2,000,000}$ or 15 times as much as the stress in the concrete. This figure 15 (or 12 or 10 according to the value of $E_c$) is the value of the modular ratio $\frac{E_s}{E_c}$, and is represented generally by $n$. This means that when concrete containing reinforcing steel is loaded, so long as the bond or adhesion between steel and concrete has not broken down and the concrete is intact, the stress in the steel will be $n$ times the stress in the concrete at that same point in the section; or when a reinforced concrete beam is bent the stress in the steel will be $n$ times the stress in the concrete at any point (in the same section) which is the same distance from the neutral axis, stress being proportional to distance from the neutral axis (see p. 8).

There are several other factors mentioned later which in a true statement of the conditions would have to be considered, but so long as the student is aware that they exist he may in the more modest designs of his early years dismiss them from his calculations—they may safely be left to worry the experts. Such factors are:

(*a*) In hardening the concrete shrinks, and adhesion between concrete and steel causes the steel to resist this shrinkage; the result is that the steel is compelled to contract in length and the concrete is "stretched" from that condition it would have been in had no reinforcement been provided. The amounts of these movements if required can be easily calculated if the relative cross-sectional areas, the value of $n$, and the free shrinkage coefficient of the concrete are all known. The stress set up between steel and concrete results

in "initial" stresses in these two materials, and stresses due to subsequent loading will be superimposed on these initial stresses.

(*b*) Due to temperature changes and the slightly differing coefficients of expansion of the two materials slight relative stresses will be set up. These are negligible.

(*c*) Due to plastic yield in the concrete, and to a very much smaller extent plastic yield in the steel, stress is not strictly proportional to strain, and the effect of the former can only be allowed for approximately by the assumption of the reduced modular ratio as referred to on p. 35.

As previously stated, the student may ignore all of these effects. There is one other factor which might seriously interfere with the results. If bond stress between steel and concrete has not been provided for and "slip" occurs, the two materials act more or less independently and the stresses cannot be computed; it is important in design to keep bond stresses reasonably low, and in construction to keep the bars free from loose rust or scale, grease or paint, and to use a reasonably dry mix so that water pockets do not collect under the bars as is the tendency with sloppy or even fairly wet mixes.

The crushing strength of concrete is generally determined by tests on cubes, the crushing strength of which is generally about $1\frac{1}{3}$ times the strength of cylinders whose diameter is equal to one-half their height. For an ordinary 1 : 2 : 4 concrete with normal conditions of curing the crushing strength of the cube should be approximately 2,400 lb. per square inch at 28 days, and a factor of safety of from 3 to 4 on this value would be used for determining the allowable working stresses. The stresses given in *Table* I were adopted by the Ministry of Transport in 1931 and are adopted for a basis of design in this book. The stresses currently acceptable by the Ministry of Transport are given in Memorandum No. 577.

TABLE I.

MINISTRY OF TRANSPORT—PERMISSIBLE CONCRETE STRESSES IN BRIDGE DESIGN (1931).

| Concrete Mix. | | | Working Stress in Flexure, $f_c$. | Modular Ratio. | Crushing strength of concrete in 6-in. cubes. | |
|---|---|---|---|---|---|---|
| Cement. | Fine Aggregate. | Coarse Aggregate. | | | At 28 days with ordinary Portland cement. At 7 days with rapid-hardening Portland cement. | An additional test (if required) as an indication, at 7 days with ordinary Portland cement, at 3 days with rapid-hardening Portland cement, should give results as below. |
| lb. | cb. ft. | cb. ft. | lb. per sq. inch. | $n$. | lb. per sq. inch. | lb. per sq. inch. |
| A | 2 | 4 | 5A + 300 | — | 15A + 900 | 10A + 600 |
| 90 | 2 | 4 | 750 | 15 | 2,250 | 1,500 |
| 120 | 2 | 4 | 900 | 15 | 2,700 | 1,800 |
| 150 | 2 | 4 | 1,050 | 12 | 3,150 | 2,100 |
| 180 | 2 | 4 | 1,200 | 10 | 3,600 | 2,400 |

The crushing strength of concrete in cubes does not exactly correspond with the calculated stresses at failure in beams and columns. In a beam the strength is from 1·0 to 1·2 × the cube strength, while for columns the strength is about 0·7 × the cube strength, plus the strength of the reinforcing steel.

For this reason the previous working stresses based on cube strength are adopted for flexure, but a figure 20 per cent. lower is adopted for the average stress in a member subjected to direct compression; a member subjected to combined bending and compression should conform to both requirements simultaneously.

Concrete is weak in tension, its resistance to stress being generally about one-tenth of its compressive strength. For this reason reinforcing steel is used in any portion of a member liable to tensile stress. And since, for economical design, the reinforcing steel should be stressed to about 18,000 lb. per square inch and the surrounding concrete therefore to $\frac{1}{n}$ or say one-fifteenth of this value, which is 1,200 lb. per square inch, we always assume the surrounding concrete to have cracked and no reliance is placed upon its tensile resistance. In practice the surrounding concrete where the steel is highly stressed has cracked, but owing to the bond between concrete and steel the cracks are well distributed and microscopic and are of no serious consequence. It is for this reason, however, that while with improved cements allowable concrete stresses are increased the allowable steel stress remains at 18,000 lb. per square inch.

The true shear strength of concrete is about three-quarters of the compressive strength, but in a beam subjected to shear diagonal tension stresses are set up comparable to the calculated shear stress, and these are responsible for "shear" failures. Working stresses in shear or diagonal tension are therefore generally limited to one-tenth of the allowable compressive stress, and when these are exceeded it becomes necessary to provide "web reinforcement" such as stirrups and bent-up bars.

Bond stress between concrete and steel should generally be kept below 100 lb. per square inch; this figure is however sometimes exceeded where effective end anchorage is provided.

We may now pass to the consideration of the action in bending of a reinforced concrete beam.

### Moment of Resistance of a Simply-Reinforced Concrete Section.

*Fig.* 33 represents the section of a reinforced concrete beam of breadth $b$ and depth $d$ from compression face to centre of gravity of reinforcing steel; the concrete below the steel does not require to be considered except for its weight. The depth to the neutral axis is a fraction termed $k$ of the depth $d$, and the distance from the neutral axis to the steel is therefore $(1 - k)d$. The other parts of *Fig.* 33 give the stress relationships between the various parts of the section, the right-hand diagram representing stress and the middle one strain. The centre of compression in the concrete corresponds with the centre of gravity of the triangle and is therefore $\frac{kd}{3}$ from the compression face, and the total compressive force taken as acting at this point is $\frac{1}{2}.f_c bkd$. Owing to reasons previously given the concrete in tension is neglected and the total tensile force will be the area of steel $A_s$ multiplied by its stress $f_s$.

The "lever arm" between these forces is $d - \frac{kd}{3}$ and this is termed $jd$, $j$

being a fraction of $d$ usually in the neighbourhood of 0·9. The area of steel $A_s$ may be taken as a fraction $p$ of the concrete area $bd$, so that

$$A_s = pbd.$$

By similar triangles
$$f_c = \frac{f_s k}{n(1-k)} \qquad (23)$$

From all of these relationships we deduce that

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (24)$$

and also

$$k = \sqrt{2pn + p^2n^2} - pn \qquad (25)$$

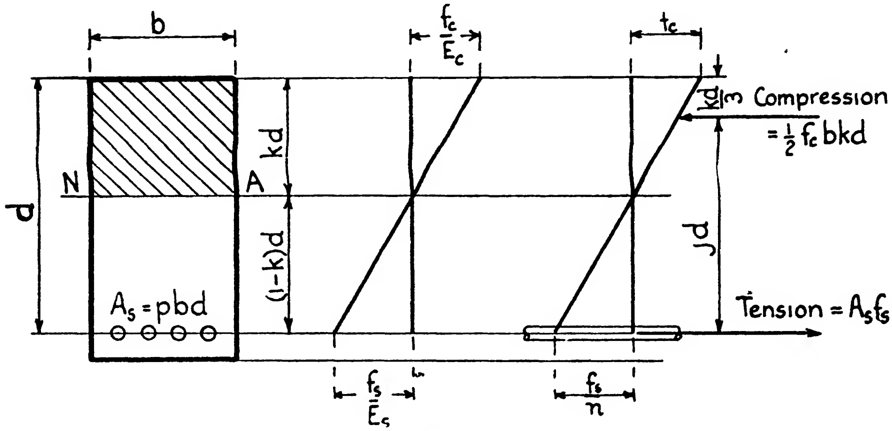



Fig. 33.

Considering the equilibrium of the vertical section in *Fig.* 33, the total horizontal compression must equal the total horizontal tension,

$$\therefore \tfrac{1}{2} f_c bkd = A_s f_s \qquad (26)$$

and also, owing to equilibrium, the internal resisting moment must balance the applied bending moment $M$,

$$\therefore M = \tfrac{1}{2} f_c bkdjd = A_s f_s jd \qquad (27)$$

If we substitute $pbd$ for $A_s$, and a constant $R$ for $f_s pj$ we get

$$M = Rbd^2 \qquad (28)$$

For fixed allowable working stresses for steel and concrete of a given modular ratio $n$, $p$ and $j$ are determined, and therefore $R$ is known. When the applied bending moment is calculated the necessary effective depth ($d$) for the section, whether it be slab or beam, may be found by equation (29) which follows directly from equation (28),

$$d = \sqrt{\frac{M}{Rb}} \qquad (29)$$

In the design of slabs a strip 1 ft. wide is investigated. The units are in

inches and inch lb., but $M$ and $b$ may for convenience both be taken together in ft. lb. and ft. units, $d$ remaining in inches.

*Table* IV, p. 83, is provided, giving the values of $R$, $p$, $j$, and $k$ corresponding to common values of concrete and steel stresses.

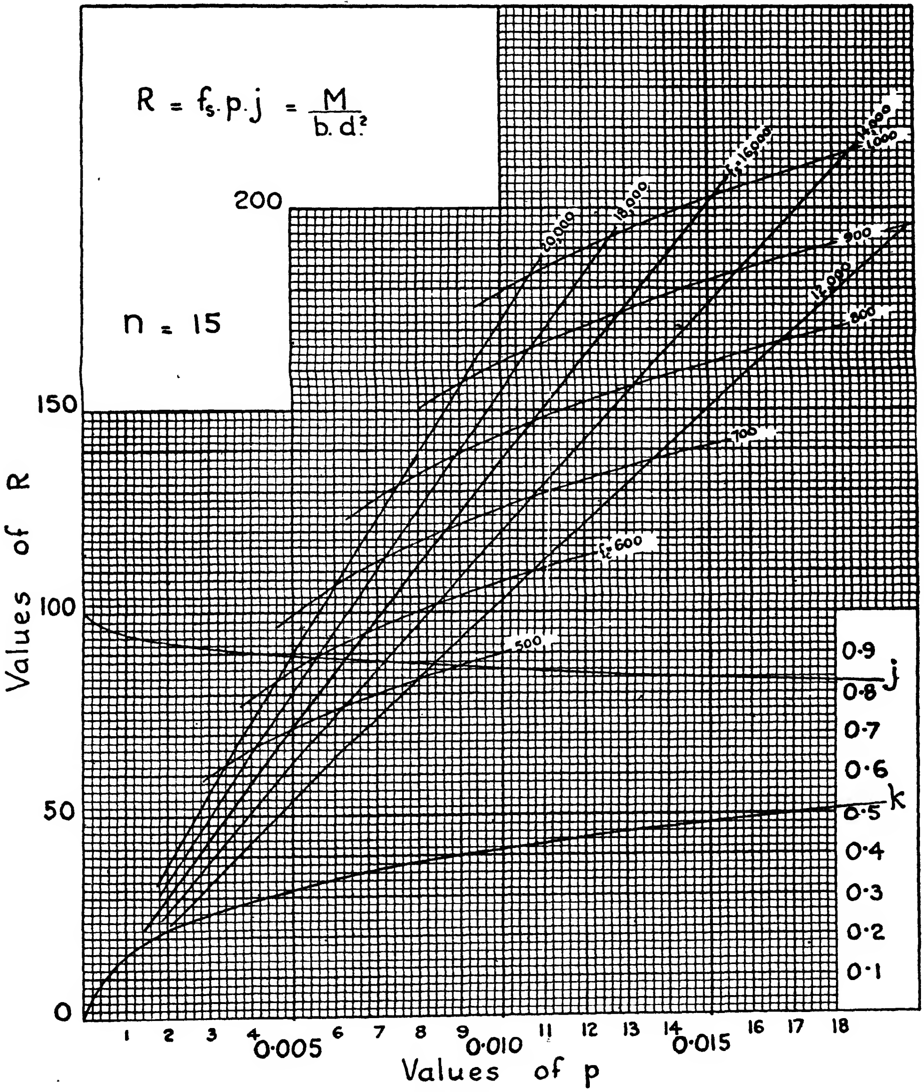



**Fig. 34A.**

Design charts may conveniently be employed to give values of $R$ and $p$ corresponding to various allowable working stresses, and these are provided for the three common modular ratios, in *Figs.* 34A, 34B, and 34C.

It is necessary to know the values of $A_s$ for numbers of bars in groups and also for various sizes and spacings of bars, and these are given for reference in *Tables* II and III. In *Table* III the term $\Sigma o$ refers to the sum of the perimeters

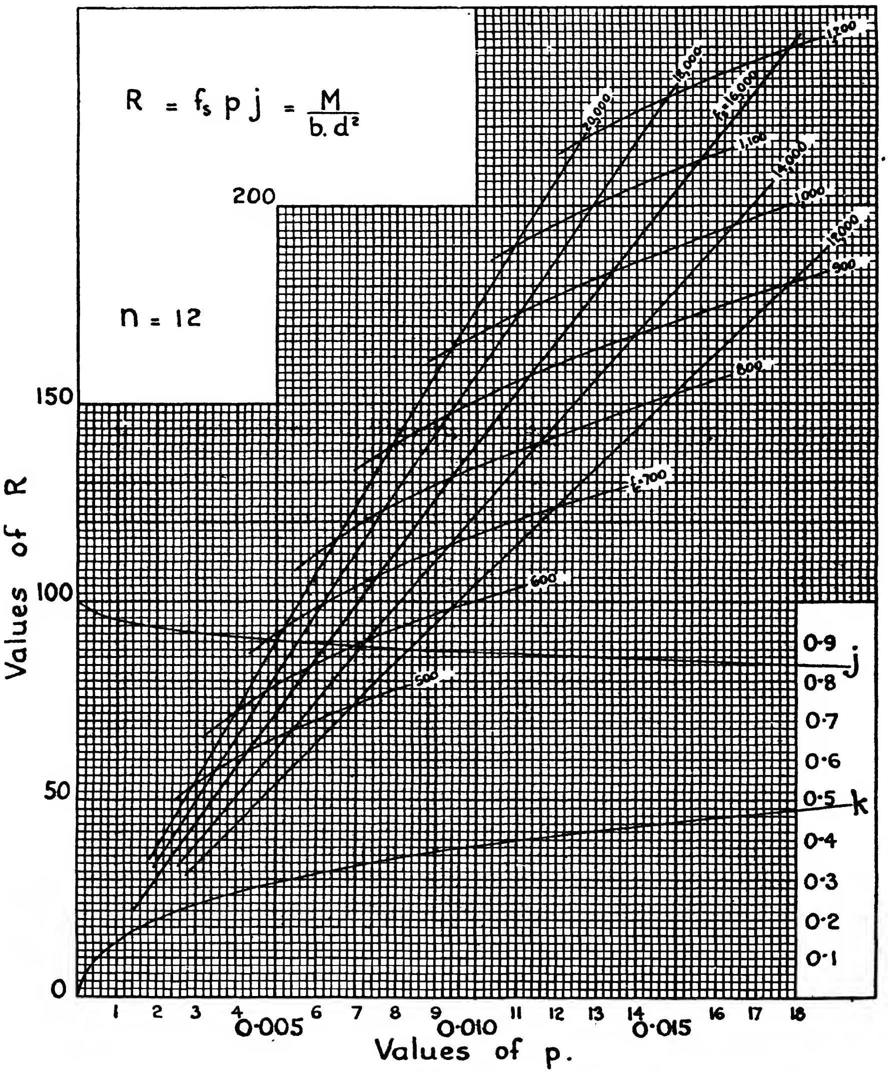



**Fig. 34B.**

of bars in a foot wide strip of slab and these figures are used, as will be explained later in the determination of bond stresses.

Examples demonstrating the use of formulæ and design charts will be given in the next chapter.

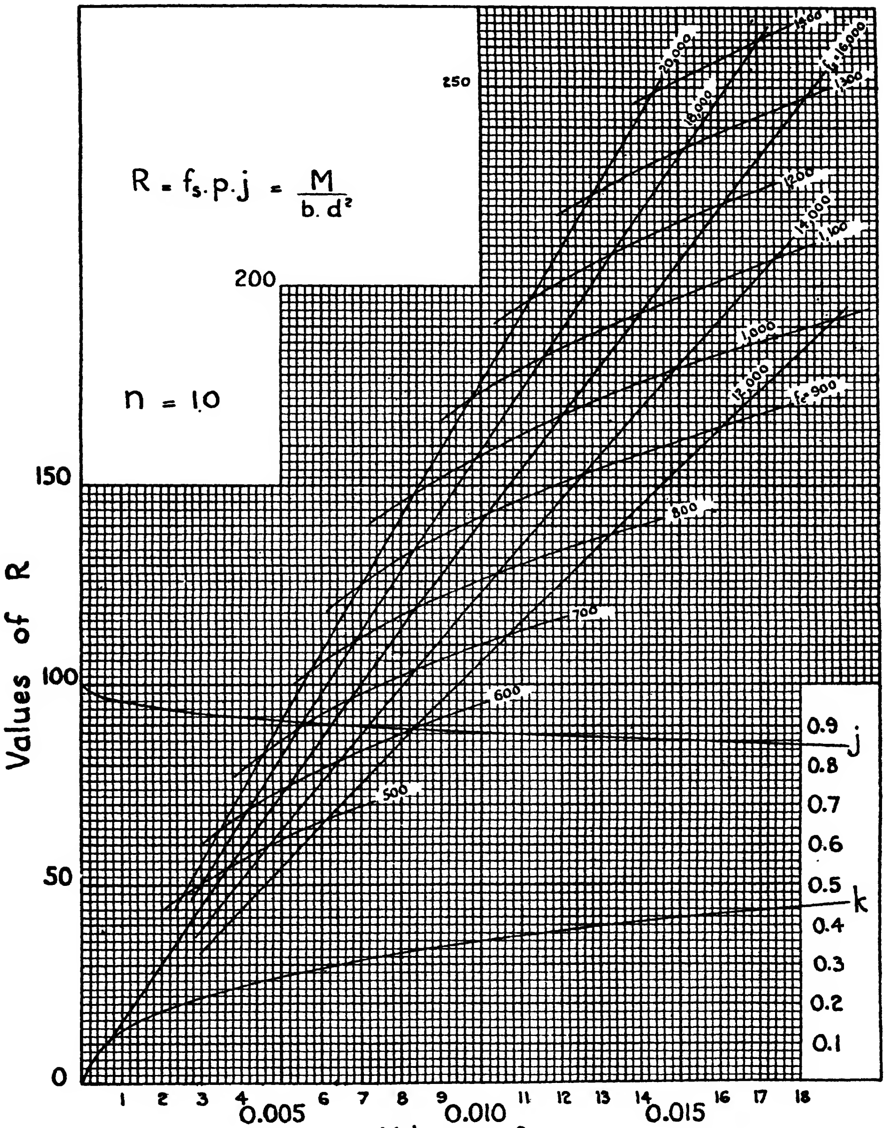

R = fs.p.j = M / b.d²
n = 10
Values of R
0
50
100
150
200
250
Values of p.
1 2 3 4 6 7 8 9 11 12 13 14 16 17 18
0.005
0.010
0.015
fs = 16,000
20,000
18,000
14,000
12,000
fc = 900
1000
1100
1200
1300
1400
800
700
600
500
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
j
k

Fig. 34c.

TABLE II.

SECTIONAL AREAS OF GROUPS OF ROUND BARS IN SQUARE INCHES.

| Diam. of bars in inches. | Number of bars. | | | | | | | | | | | | | | Diam. of bars in inches. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | |
| $\frac{1}{4}$ | 0·049 | ·098 | ·147 | ·196 | ·245 | ·294 | ·343 | ·392 | ·441 | ·491 | ·540 | ·589 | ·638 | ·687 | $\frac{1}{4}$ |
| $\frac{5}{16}$ | 0·076 | ·153 | ·230 | ·306 | ·383 | ·460 | ·536 | ·613 | ·690 | ·767 | ·843 | ·920 | ·997 | 1·073 | $\frac{5}{16}$ |
| $\frac{3}{8}$ | 0·110 | ·220 | ·331 | ·441 | ·552 | ·662 | ·772 | ·883 | ·993 | 1·104 | 1·214 | 1·324 | 1·435 | 1·545 | $\frac{3}{8}$ |
| $\frac{7}{16}$ | 0·150 | ·300 | ·450 | ·601 | ·751 | ·901 | 1·052 | 1·202 | 1·352 | 1·503 | 1·653 | 1·803 | 1·953 | 2·104 | $\frac{7}{16}$ |
| $\frac{1}{2}$ | 0·196 | ·392 | ·588 | ·785 | ·981 | 1·177 | 1·374 | 1·570 | 1·766 | 1·963 | 2·159 | 2·355 | 2·551 | 2·748 | $\frac{1}{2}$ |
| $\frac{9}{16}$ | 0·248 | ·497 | ·745 | ·994 | 1·242 | 1·491 | 1·739 | 1·988 | 2·236 | 2·485 | 2·733 | 2·982 | 3·230 | 3·479 | $\frac{9}{16}$ |
| $\frac{5}{8}$ | 0·306 | ·613 | ·920 | 1·227 | 1·534 | 1·840 | 2·147 | 2·454 | 2·761 | 3·068 | 3·374 | 3·681 | 3·988 | 4·295 | $\frac{5}{8}$ |
| $\frac{11}{16}$ | 0·371 | ·742 | 1·113 | 1·484 | 1·856 | 2·227 | 2·598 | 2·969 | 3·340 | 3·712 | 4·083 | 4·454 | 4·825 | 5·196 | $\frac{11}{16}$ |
| $\frac{3}{4}$ | 0·441 | ·883 | 1·325 | 1·767 | 2·209 | 2·650 | 3·092 | 3·534 | 3·976 | 4·418 | 4·859 | 5·301 | 5·743 | 6·185 | $\frac{3}{4}$ |
| $\frac{13}{16}$ | 0·518 | 1·037 | 1·555 | 2·074 | 2·592 | 3·111 | 3·629 | 4·148 | 4·665 | 5·185 | 5·703 | 6·222 | 6·740 | 7·259 | $\frac{13}{16}$ |
| $\frac{7}{8}$ | 0·601 | 1·202 | 1·803 | 2·405 | 3·006 | 3·607 | 4·209 | 4·810 | 5·411 | 6·013 | 6·614 | 7·215 | 7·816 | 8·418 | $\frac{7}{8}$ |
| $\frac{15}{16}$ | 0·690 | 1·380 | 2·070 | 2·761 | 3·451 | 4·141 | 4·832 | 5·522 | 6·212 | 6·903 | 7·593 | 8·283 | 8·973 | 9·664 | $\frac{15}{16}$ |
| 1 | 0·785 | 1·570 | 2·356 | 3·141 | 3·927 | 4·712 | 5·497 | 6·285 | 7·068 | 7·854 | 8·639 | 9·424 | 10·210 | 10·995 | 1 |
| $1\frac{1}{16}$ | 0·886 | 1·772 | 2·659 | 3·545 | 4·432 | 5·318 | 6·204 | 7·091 | 7·977 | 8·864 | 9·750 | 10·636 | 11·523 | 12·409 | $1\frac{1}{16}$ |
| $1\frac{1}{8}$ | 0·994 | 1·988 | 2·982 | 3·976 | 4·970 | 5·964 | 6·958 | 7·952 | 8·946 | 9·940 | 10·934 | 11·928 | 12·922 | 13·916 | $1\frac{1}{8}$ |
| $1\frac{3}{16}$ | 1·107 | 2·215 | 3·322 | 4·430 | 5·537 | 6·645 | 7·752 | 8·860 | 9·967 | 11·075 | 12·182 | 13·290 | 14·397 | 15·505 | $1\frac{3}{16}$ |
| $1\frac{1}{4}$ | 1·227 | 2·454 | 3·681 | 4·908 | 6·136 | 7·363 | 8·590 | 9·817 | 11·044 | 12·272 | 13·499 | 14·726 | 15·953 | 17·180 | $1\frac{1}{4}$ |
| $1\frac{5}{16}$ | 1·353 | 2·706 | 4·059 | 5·412 | 6·765 | 8·118 | 9·471 | 10·824 | 12·177 | 13·530 | 14·883 | 16·236 | 17·589 | 18·942 | $1\frac{5}{16}$ |
| $1\frac{3}{8}$ | 1·484 | 2·969 | 4·454 | 5·939 | 7·424 | 8·909 | 10·394 | 11·879 | 13·364 | 14·849 | 16·333 | 17·818 | 19·303 | 20·788 | $1\frac{3}{8}$ |
| $1\frac{7}{16}$ | 1·623 | 3·246 | 4·869 | 6·492 | 8·115 | 9·738 | 11·361 | 12·984 | 14·607 | 16·230 | 17·853 | 19·476 | 21·099 | 22·722 | $1\frac{7}{16}$ |
| $1\frac{1}{2}$ | 1·767 | 3·534 | 5·301 | 7·068 | 8·835 | 10·602 | 12·369 | 14·136 | 15·903 | 17·671 | 19·438 | 21·205 | 22·972 | 24·739 | $1\frac{1}{2}$ |
| $1\frac{9}{16}$ | 1·917 | 3·836 | 5·752 | 7·670 | 9·587 | 11·505 | 13·422 | 15·340 | 17·257 | 19·175 | 21·092 | 23·010 | 24·927 | 26·845 | $1\frac{9}{16}$ |
| $1\frac{5}{8}$ | 2·073 | 4·147 | 6·221 | 8·295 | 10·369 | 12·443 | 14·517 | 16·591 | 18·665 | 20·739 | 22·812 | 24·886 | 26·960 | 29·034 | $1\frac{5}{8}$ |
| $1\frac{11}{16}$ | 2·236 | 4·473 | 6·709 | 8·946 | 11·182 | 13·419 | 15·655 | 17·892 | 20·128 | 22·365 | 24·601 | 26·838 | 29·074 | 31·311 | $1\frac{11}{16}$ |
| $1\frac{3}{4}$ | 2·405 | 4·810 | 7·215 | 9·621 | 12·026 | 14·431 | 16·837 | 19·242 | 21·647 | 24·053 | 26·458 | 28·863 | 31·268 | 33·674 | $1\frac{3}{4}$ |
| $1\frac{13}{16}$ | 2·580 | 5·160 | 7·740 | 10·320 | 12·901 | 15·481 | 18·061 | 20·641 | 23·221 | 25·802 | 28·382 | 30·962 | 33·542 | 36·122 | $1\frac{13}{16}$ |
| $1\frac{7}{8}$ | 2·761 | 5·522 | 8·283 | 11·044 | 13·086 | 16·567 | 19·328 | 22·089 | 24·850 | 27·612 | 30·375 | 33·134 | 35·895 | 38·656 | $1\frac{7}{8}$ |
| $1\frac{15}{16}$ | 2·948 | 5·896 | 8·844 | 11·793 | 14·741 | 17·689 | 20·638 | 23·586 | 26·534 | 29·483 | 32·431 | 35·379 | 38·327 | 41·276 | $1\frac{15}{16}$ |
| 2 | 3·141 | 6·283 | 9·424 | 12·566 | 15·708 | 18·849 | 21·991 | 25·132 | 28·274 | 31·416 | 34·557 | 37·699 | 40·840 | 43·982 | 2 |

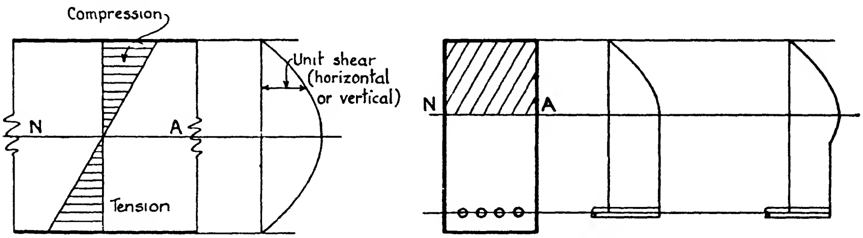
TABLE III.

AREAS, WEIGHTS, CIRCUMFERENCES AND SPACING OF ROUND BARS.

| Size in inches. | Area in sq. inches. | Weight per ft. lb. | | Area of steel and Σo per foot width spaced as follows: | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 3 in. | 3½ in. | 4 in. | 4½ in. | 5 in. | 5½ in. | 6 in. | 6½ in. | 7 in. | 7½ in. | 8 in. | 8½ in. | 9 in. | 10 in. | 11 in. | 12 in. | 15 in. |
| ¼ | ·0491 | ·167 | Σo | 3·14 | 2·69 | 2·35 | 2·09 | 1·88 | 1·71 | 1·57 | 1·45 | 1·35 | 1·26 | 1·18 | 1·11 | 1·05 | ·94 | ·86 | ·79 | ·63 |
| | | | Area | ·20 | ·17 | ·15 | ·13 | ·12 | ·11 | ·10 | ·09 | ·08 | ·08 | ·07 | ·07 | ·07 | ·06 | ·05 | ·05 | ·04 |
| ⅜ | ·1104 | ·376 | Σo | 4·72 | 4·03 | 3·53 | 3·15 | 2·83 | 2·57 | 2·36 | 2·19 | 2·02 | 1·88 | 1·77 | 1·67 | 1·57 | 1·41 | 1·28 | 1·18 | ·95 |
| | | | Area | ·44 | ·38 | ·33 | ·29 | ·26 | ·25 | ·22 | ·20 | ·19 | ·18 | ·17 | ·16 | ·15 | ·13 | ·12 | ·11 | ·09 |
| ½ | ·1963 | ·668 | Σo | 6·30 | 5·37 | 4·71 | 4·20 | 3·77 | 3·42 | 3·14 | 2·92 | 2·67 | 2·51 | 2·36 | 2·22 | 2·08 | 1·88 | 1·71 | 1·57 | 1·27 |
| | | | Area | ·78 | ·67 | ·59 | ·52 | ·47 | ·43 | ·39 | ·36 | ·34 | ·31 | ·29 | ·28 | ·26 | ·24 | ·21 | ·20 | ·16 |
| ⅝ | ·3068 | 1·043 | Σo | 7·85 | 6·72 | 5·89 | 5·24 | 4·71 | 4·28 | 3·93 | 3·63 | 3·36 | 3·14 | 2·93 | 2·78 | 2·61 | 2·36 | 2·14 | 1·96 | 1·57 |
| | | | Area | 1·23 | 1·05 | ·92 | ·82 | ·74 | ·67 | ·61 | ·57 | ·52 | ·49 | ·46 | ·43 | ·41 | ·37 | ·33 | ·31 | ·25 |
| ¾ | ·4418 | 1·502 | Σo | 9·42 | 8·06 | 7·07 | 6·29 | 5·65 | 5·14 | 4·71 | 4·36 | 4·05 | 3·77 | 3·53 | 3·32 | 3·13 | 2·83 | 2·57 | 2·36 | 1·89 |
| | | | Area | 1·77 | 1·51 | 1·33 | 1·18 | 1·06 | ·96 | ·88 | ·82 | ·76 | ·71 | ·66 | ·63 | ·59 | ·53 | ·48 | ·44 | ·35 |
| ⅞ | ·6013 | 2·044 | Σo | 11·00 | 9·40 | 8·20 | 7·40 | 6·60 | 6·00 | 5·50 | 5·10 | 4·70 | 4·40 | 4·10 | 3·88 | 3·80 | 3·30 | 3·00 | 2·75 | 2·20 |
| | | | Area | 2·40 | 2·06 | 1·80 | 1·61 | 1·44 | 1·31 | 1·20 | 1·11 | 1·03 | ·96 | ·90 | ·85 | ·80 | ·72 | ·66 | ·60 | ·48 |
| 1 | ·7854 | 2·670 | Σo | 12·56 | 10·70 | 9·40 | 8·40 | 7·50 | 6·80 | 6·28 | 5·80 | 5·40 | 5·00 | 4·70 | 4·44 | 4·20 | 3·80 | 3·40 | 3·14 | 2·52 |
| | | | Area | 3·14 | 2·69 | 2·36 | 2·09 | 1·88 | 1·71 | 1·57 | 1·45 | 1·35 | 1·26 | 1·18 | 1·11 | 1·05 | ·94 | ·86 | ·79 | ·63 |
| 1⅛ | ·9940 | 3·380 | Σo | 14·12 | 12·10 | 10·60 | 9·40 | 8·50 | 7·70 | 7·10 | 6·50 | 6·00 | 5·70 | 5·30 | 4·99 | 4·70 | 4·20 | 3·90 | 3·53 | 2·83 |
| | | | Area | 3·98 | 3·41 | 2·98 | 2·65 | 2·39 | 2·17 | 1·99 | 1·84 | 1·70 | 1·59 | 1·49 | 1·40 | 1·33 | 1·19 | 1·08 | ·99 | ·79 |
| 1¼ | 1·2272 | 4·172 | Σo | 15·70 | 13·40 | 11·80 | 10·50 | 9·40 | 8·60 | 7·87 | 7·30 | 6·70 | 6·30 | 5·90 | 5·55 | 5·20 | 4·70 | 4·30 | 3·93 | 3·14 |
| | | | Area | 4·91 | 4·21 | 3·68 | 3·27 | 2·95 | 2·68 | 2·45 | 2·27 | 2·10 | 1·96 | 1·84 | 1·73 | 1·64 | 1·47 | 1·34 | 1·23 | ·99 |
| 1⅜ | 1·4849 | 5·049 | Σo | 17·28 | 14·81 | 12·96 | 11·52 | 10·38 | 9·43 | 8·65 | 7·98 | 7·40 | 6·92 | 6·48 | 6·10 | 5·76 | 5·18 | 4·72 | 4·32 | 3·46 |
| | | | Area | 5·94 | 5·09 | 4·45 | 3·96 | 3·56 | 3·24 | 2·97 | 2·74 | 2·55 | 2·38 | 2·23 | 2·10 | 1·98 | 1·78 | 1·62 | 1·48 | 1·19 |
| 1½ | 1·7671 | 6·008 | Σo | 18·83 | 16·14 | 14·12 | 12·55 | 11·30 | 10·27 | 9·42 | 8·70 | 8·07 | 7·54 | 7·06 | 6·65 | 6·28 | 5·65 | 5·14 | 4·71 | 3·76 |
| | | | Area | 7·07 | 6·06 | 5·30 | 4·71 | 4·24 | 3·86 | 3·53 | 3·27 | 3·03 | 2·83 | 2·65 | 2·50 | 2·36 | 2·12 | 1·93 | 1·77 | 1·42 |

## Shear Stresses in a Reinforced Concrete Section.

The distribution of shear stress (both horizontal and vertical) for a homogeneous beam is shown in *Fig.* 35 (*a*). The difference in the present case is that tension below the neutral axis is assumed to be taken entirely by the steel, and on this assumption the horizontal shear stress (and therefore the vertical shear stress also) remains constant from the neutral axis to the steel where the whole of its work has to be performed through the agency of the bond stress (which is in fact the horizontal shear stress at the final stage) on the reinforcing bars. In practice the whole of the concrete below the neutral axis cannot be said to have




(*a*) Homogeneous Beam (*b*) Reinforced Concrete Beam.

Fig. 35.

cracked, and tension is developed in it by the action of the horizontal shear, and gradually as its work is performed the shear stress intensity becomes reduced in the manner shown in *Fig.* 35 (*a*). However, there comes a stage where the concrete has cracked in tension, and from this point down the horizontal shear remains constant as shown in the right-hand portion of *Fig.* 35 (*b*). On the theoretical assumption that the concrete sustains no tension we have the condition represented in the middle portion of *Fig.* 35 (*b*). Accepting this condition we note that

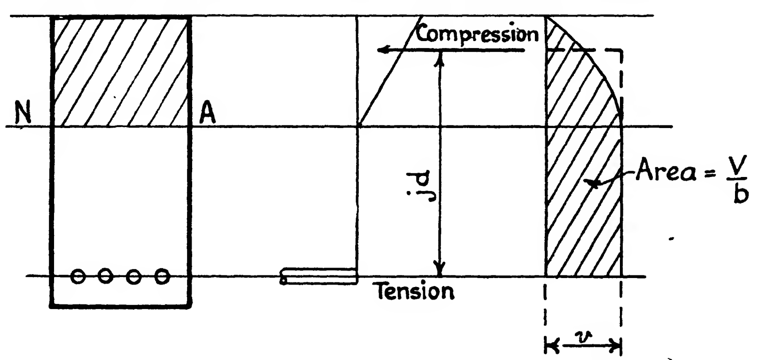



Fig. 36.

the area of the semi-parabola above the neutral axis is $\frac{2}{3}$ × base × height, so that if it is replaced by an equivalent rectangle of width equal to that of the rectangle below the neutral axis, the top of the rectangle will coincide with the centre of compression (see *Fig.* 36) distant $\frac{kd}{3}$ below the compression face, and the area of the shear stress diagram is therefore $vjd$ where $v$ is the maximum unit shear.

The shear operates on a section of width $b$, so that the total internal shear equals $vbjd$, and this is equal to the external shear, $V$.

$$\therefore v = \frac{V}{b \times jd} \qquad (30)$$

As previously stated it is permissible in design to allow $v$ to equal up to one-tenth of the allowable compressive stress without providing web reinforcement, although in practice beams are seldom constructed without some system of stirrups. When this value is exceeded web reinforcement should be provided as explained below.

### Diagonal Tension.

*Fig.* 37 represents an elementary cube taken from close to the neutral plane of a loaded beam. At this position the direct fibre stresses are zero, and the horizontal and vertical shears on this cube combine to form tensile and compressive forces in the 45-deg. planes mutually at right angles, deforming the cube so that it becomes lozenge shape (*Fig.* 37). Away from the neutral plane the

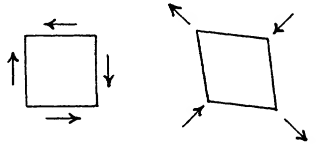


**Fig. 37.**

primary fibre stresses, tension or compression, combine with these diagonal stresses and the directions of the resulting forces are a little indefinite.

It is the diagonal tension resulting from a combination of the primary tensile stresses and the shear stresses which is responsible for so-called "shear failures." Experiment has shown that shear stress as calculated is a good measure of diagonal tension, and web reinforcement is calculated from the shear.

The assumption that concrete will not resist tension applies only to the primary stresses where the concrete has almost certainly cracked, and is not applied to diagonal tension when this stress is low. When this stress is high, or when the shear stress, $v$, exceeds say 120 lb. per square inch, the whole shear (diagonal tension) is provided for by web reinforcement; below this figure we may reasonably assume the concrete to take one-third of the shear, and web reinforcement is provided for two-thirds.

Beams are considerably strengthened by a good system of web reinforcement: this takes the following forms:

(1) Vertical stirrups,

(2) Bent-up bars, and

(3) A combination of bent-up bars and stirrups.

The third method is preferable in all cases where the shear stress is high, and with it $v$, as calculated by equation (30), may be taken as high as 160 to 220 lb. per square inch depending upon the proportions of the concrete; it is also, of course, suitable with the lower shear stresses. Methods (1) and (2) are suitable when $v$ is below 120, but it is generally a little difficult to find bars available for the protection of all sections of a beam by method (2) alone.

A typical shear failure is represented in *Fig.* 38. The crack crosses the neutral plane roughly at 45 deg. and below and above this position its direction is changed as explained on p. 47.

The exact manner in which stirrups act is a little uncertain, but they are generally spaced by formula (31) which is derived from *Fig.* 38 as follows:

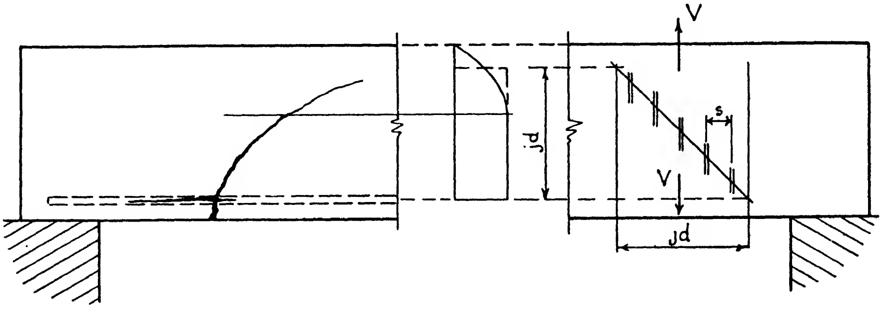



**Fig. 38.**

A 45-deg. plane extending from the centre of tension (the reinforcing steel) to the centre of compression includes a length of beam $jd$. Assuming a diagonal tension failure to threaten in this plane, and assuming the vertical forces tending to produce this separation to be equal to the average shear $V$, this force must be resisted by the vertical stirrups provided in this length of beam. The number of stirrups is $\frac{jd}{s}$, and each carries a load of $A_s f_s$ lb., where $A_s$ is the total cross-sectional area of each stirrup.

$$\therefore\ V = A_s f_s \frac{jd}{s}, \text{ where } s \text{ is the spacing,}$$

$$\text{or, } s = \frac{A_s f_s jd}{V} \qquad \ldots \qquad (31)$$

When it is assumed that one-third of the shear is taken on the concrete $V_s$ would be substituted for $V$, where $V_s = \frac{2}{3}V$.

The allowable value for $f_s$ in stirrups is sometimes taken as 18,000 lb. per square inch, but this practice is probably inadvisable when $v$ approaches the maximum value allowable; for normal values it is safe to take $f_s = 16{,}000$ lb. per square inch. Where this is done and $j$ is assumed to be 0·875, formula (31) becomes

$$s = \frac{14{,}000\ A_s d}{V} \qquad \ldots \qquad (31\text{A})$$

Bars bent up at 45 deg. lie in the planes of principal tension where these cross the neutral plane. Bars bent up at 30 deg. or thereabouts lie in the planes of principal tension a little below this position, and are fully as effective: they have the additional advantage that the reduction in main tensile reinforcement necessitated by bending up these bars, and justifiable by reason of the reduction in the bending moment towards the supports, is made gradually, and the bend can be made with a larger radius and less crushing effect on the concrete.

Since the vertical stirrups pass through the planes of principal tension at an angle (assumed at 45 deg.) they are not fully efficient as web reinforcement, and the component of their stress along the planes of principal tension is only cos 45 deg. (approximately 0·7) of the stirrup stress. Bent-up bars which lie in the planes of principal tension may be assumed to be fully efficient, and their spacing can be obtained in the same way as for vertical stirrups by introducing this efficiency factor of $\frac{1}{\cos 45 \text{ deg.}}$ or sec 45 deg. If cos 45 deg. is taken approximately as 0·7 the reciprocal becomes 1·43.

The formula for spacing bent-up bars therefore becomes

$$s = \frac{1{\cdot}43 \times A_s f_s jd}{V} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

With $f_s$ equal to 16,000 lb. per square inch, and $j$ equal to 0·875 this becomes

$$s = \frac{20{,}000\, A_s d}{V} \quad . \quad . \quad . \quad . \quad . \quad (32\text{A})$$

Experiment has shown that, regardless of the formula, a wide spacing should never be adopted: the spacing of vertical stirrups should be limited to $\frac{2}{3}d$, and that of bent-up bars to $\frac{3}{4}d$, measured along the neutral plane. Bars should always be bent up in pairs symmetrically placed in the beam cross section, and at least two pairs should be bent up if they are to be relied upon as shear reinforcement. Bent-up bars should extend " bond distance " (see later) above the neutral plane.

When both bent-up bars and vertical stirrups are employed as web reinforcement, the method of calculation should be as follows: first space the bent-up bars with a distance of $\frac{2}{3}d$ or $\frac{3}{4}d$ measured along the neutral plane between each pair, or closer if necessary to meet the requirements of bending moment on the main tensile bars in the bottom of the beam; calculate the shear which the bars thus spaced are capable of resisting efficiently—the formula follows from formula (32),

$$V_1 = \frac{1{\cdot}43 \times A_s f_s jd}{s} \quad . \quad . \quad . \quad . \quad . \quad (33)$$

The shear governing the stirrup spacing will then be $V - V_1$. *Fig.* 39 shows typical web reinforcement by a combination of stirrups and bent-up bars. The portion of the beam close to the support is protected by bars bent up at 45 deg., while the other bars are bent up at an angle of about 30 deg.

### Bond Stresses.

It has been explained that fibre stress is produced by horizontal shear, and that when reinforcing bars are used the stress in them is produced by the action of horizontal shear between concrete and steel; this horizontal shear between the two materials is termed bond. It is made possible by the joint forces of adhesion and friction due to the " shrinking on " of the concrete; of these two the second is probably the greater. Both are destroyed if the steel is greasy or is painted.

The horizontal shear intensity at any position below the neutral plane is $v$

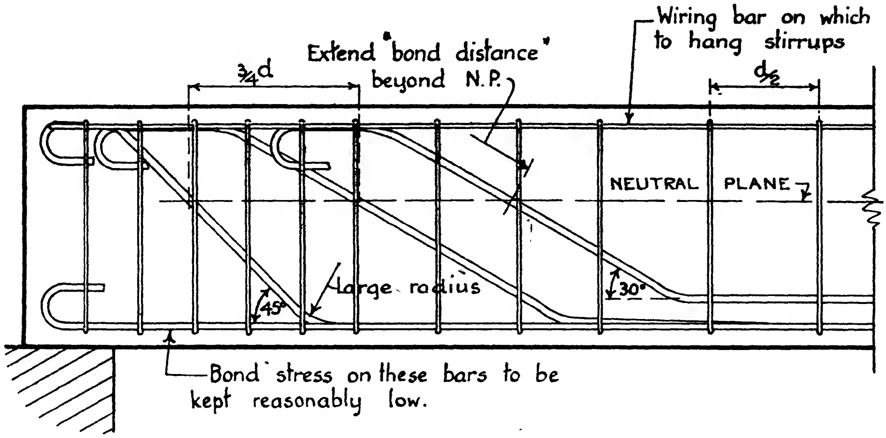



**Fig. 39.**

(*Figs.* 35 and 36). Consider a horizontal section of unit length (1 in.) at the position of the dotted line $XX$ in *Fig.* 40. The total shear on this plane will be $vb$. The whole of this stress is to be transferred to the bars through their surface area, which in a unit length is $\Sigma o$. If the unit bond stress is $u$ the total bond stress is $u\Sigma o$, from which $u\Sigma o = vb$. Substitute $\dfrac{V}{b \times jd}$ for $v$ as given in equation (30), and we get

$$u = \frac{V}{\Sigma ojd}. \qquad (34)$$

Since bars of small diameter afford a greater ratio of surface area to cross-sectional area than larger bars, it is preferable wherever bond stresses are high to supply bars of small diameter.

Referring again to *Fig.* 40 it will be noticed that the bond stress for the under-

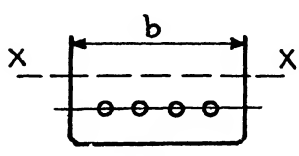



**Fig. 40.**

sides of the bars has to be transferred in horizontal shear by way of the concrete between the bars, and the spaces between the bars should never be made too small; they should always exceed the maximum size of aggregate, and should never be less than 1 in., or the diameter of the bar, whichever is the greater. A good practice adopted by many designers is to limit the minimum space to $1\frac{1}{2}$ bar diameters. The surface area of the lower half of a bar is $\dfrac{\pi d}{2} = 1{\cdot}57d$, and if the clear space between bars is made $1\frac{1}{2}d$ the horizontal shear in the concrete at the lowest layer of bars is then approximately equal to the bond stress.

Where bond stress is high near the ends of bars end anchorage may be provided by using a hook of the dimensions shown in *Fig.* 41. This is preferable to a right-angle bend which tends to split or spall off the concrete. A short transverse bar is sometimes placed inside the hook where the anchorage requirement is high. Equation (34) shows that bond stress is directly proportional to shear, and it is therefore highest at those points where the shear is a maximum such as near the supports of beams. Shear failures in simple beams are usually found to occur a short distance away from the support: the failure generally originates in "bond" failure by a local slipping of the bar in the concrete. Near the support the tensile fibre stresses due to bending moment are low and the concrete does not crack, so that the unit shear stresses are more nearly like those shown in

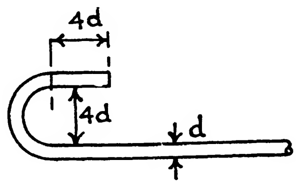



**Fig. 41.**

*Fig.* 35 (*a*), and although $V$ is high $u$ is much lower than calculated, equation (34) being based on a horizontal shear distribution such as is shown in *Fig.* 35 (*b*). The real value of $u$ more nearly approaches the calculated value at a short distance away from the support where the steel stress is higher and where the surrounding concrete has cracked. This is rightly disregarded in design, and $u$ should be based upon the value for $V$ at all sections.

Bond stresses and shear are usually low in freely-supported slabs and in cantilever walls. The allowable value for bond stress is generally from one-tenth to one-sixth of the allowable compressive strength of the concrete, but this may be exceeded close to a hook provided for end anchorage. This value may also be exceeded where the surrounding concrete is in compression, as the effect of the longitudinal compression of the concrete is to increase the transverse pressure of the concrete on the bar.

"Bond distance" is the term applied to the length of bar necessary theoretically to develop by bond the full tensile strength of the bar. It is arrived at as follows: Taking a bar of diameter $d$, the cross-sectional area is $\frac{\pi d^2}{4}$, and the perimeter is $\pi d$. The allowable direct stress is therefore $f_s \times \frac{\pi d^2}{4}$, and if the necessary length is $l$ the total bond stress is $ul\pi d$. Equating these two terms we deduce that

$$l = \frac{f_s d}{4u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (35)$$

If $f = 16{,}000$ lb. per square inch and $u = 100$ lb. per square inch,

$$= 40d \quad . \quad . \quad . \quad . \quad . \quad . \qquad (35\text{A})$$

Since the value of $u$ varies from section to section, equation (35), which is

based on a value of $u$ constant over length $l$, has not a thoroughly sound basis, but in practice it is generally found to be satisfactory and the formula is almost universally adopted. When bars are spliced they are lapped " bond distance," and when fully stressed in tension at such a splice they should also be hooked at their ends as an extra precaution.

It is not always possible to prevent a small amount of local slip, and this is likely to occur where a hook really functions as an anchorage ; if the hook takes stress from an adjacent bar the latter must lose stress suddenly, and the local bond stress must be high. A small amount of local slip where the bar is well anchored may not be detrimental, but where the slip can become progressive general failure may ensue.

## CHAPTER V

## DESIGN OF SIMPLY-SUPPORTED SLABS

THERE are several little points which the beginner may glance at before proceeding to the remaining portion of the book which deals with the application of the knowledge which, it is to be hoped, he has gained from the previous chapters.

The designer should always have clearly in his mind just what is happening to a slab or beam. He should have a mental picture of the process, and draw sketches from time to time in order to make his calculations clear to himself and easy for anyone else to check. A clear idea must precede any calculation if the result is to be of any use.

In dealing with slabs such as in a floor or a retaining wall it is usual for simplicity to consider a strip 1 ft. wide instead of working with the full width of the unit.*

Reinforced concrete weighs about 150 lb. per cubic foot, and in massive sections this figure is used in design: in thinner sections however it is simpler, and the error involved is not great, to assume 144 lb. to the cubic foot, which means that a stick of concrete 1 sq. in. in cross section and 12 in. long weighs 1 lb.; in this way the weight of a concrete slab $t$ inches thick will be $12t$ lb. per square foot.

The effective depth $d$ of a slab or beam is the dimension from the compression face to the centre of gravity of the reinforcing steel.

The effective span of a freely-supported slab or beam is taken as the clear span + the effective depth, or where the supports are narrow, as the clear span + the average width of the support; the smaller value of these alternatives is used.

The effective span of a continuous slab, monolithic with comparatively solid supporting members as in T-beam construction, is taken as the clear span between beams. The same method may be adopted with continuous beams when the supports are solid, but where the latter consist of comparatively flexible columns the span from centre to centre of the columns should be used.

Where the slab or beam is not monolithic with the support, as for example when a reinforced concrete slab is supported on the top flanges of rolled steel joists, the effective span should be calculated as for freely-supported beams.

In selecting the spacing for reinforcing bars in a slab, a close spacing such as must be adopted for beams is unsuitable; room should be allowed for workmen to step between the bars, and 4 in. is about the minimum spacing. On the other hand the spacing should not be too wide, the maximum being generally about equal to the slab thickness $t$, or in special circumstances $1\frac{1}{2}d$.

Since it will be necessary frequently to convert inches to decimals of a foot

* The condition must be such that the loading is satisfactorily distributed and the 1-ft. wide strip is a representative one. In exceptional cases with concentrated loads the load distribution must be specially considered.

the following table should be memorised. (This is accurate to the nearest second place of decimals.)

| In | Ft. |
|---|---|
| $\frac{1}{8}$ | 0·01 |
| $\frac{1}{4}$ | 0·02 |
| $\frac{1}{2}$ | 0·04 |
| 1 | 0·08 |
| 2 | 0·17 |
| 3 | 0·25 |
| 4 | 0·33 |
| 5 | 0·42 |
| 6 | 0·50 |
| 7 | 0·58 |
| 8 | 0·67 |
| 9 | 0·75 |
| 10 | 0·83 |
| 11 | 0·92 |

A factor of safety has been used to fix " working stresses " at roughly one-quarter of the ultimate strength. The dead load of a structure can be accurately determined, but the design live load may upon special occasions be exceeded. The factor of safety of the structure as a whole in such circumstances will depend on the relative value of the dead load and live load stresses. When the live load is comparable to the dead load, or exceeds it, the risk of an increase in the live load may need consideration; where, however, the live load is much smaller than the dead load, as is the case for instance in very long span bridges, a moderate increase in the live load may perhaps be insignificant. It is a good habit in the design calculations to record both dead and live load moments and shears in the manner shown in subsequent calculations in this book.

Calculations are made with the aid of the slide rule and the figures resulting from this method are quite satisfactory.

In the design of any structure we must proceed by steps in the following order:

(1) Determine all the external or applied forces, and conditions,

(2) Determine the internal moments and shears developed by all possible combinations of external forces and conditions,

(3) Calculate the stresses these produce in the members, and make the members strong enough for their work,

(4) Provide in the working drawings for the conditions assumed in design, or anticipate in the design stage any likely variations from normal conditions (an example of this is found in the design of the footpath of a bridge: the footpath is designed for pedestrian loading of perhaps 80 to 100 lb. per square foot, but the effect of the wheel of a road vehicle coming on the path should be considered and any serious results should be provided against),

(5) Detail the working drawings accurately and thoroughly.

The applied forces consist first of the dead load (which must at first be estimated as its effect will help to determine the section required), secondly of ordinary live loads to support which the structure is provided, and thirdly such forces as wind and snow: these last-named are in ordinary and simple structures generally not of serious consequence, and lie outside the scope of this book. The ordinary live loads are usually quite definitely laid down.

The determination of the internal resisting moments and shears, and of the stresses which produce them, has been explained in Chapters II and IV.

The simplest member to design is a slab freely supported on two opposite sides so that the slab spans in one direction. The bending moment is positive over the whole span, and the slab is reinforced by straight parallel bars in the bottom face. The shear is generally low and no shear reinforcement is required.

EXAMPLE.—Design a reinforced concrete slab 4 ft. 6 in. wide to span between two buildings 14 ft. $4\frac{1}{2}$ in. clear apart, the supporting walls each being 18 in. thick. A wearing surface of asphalt 1 in. thick is to be used on the slab, and the live load may be taken as 100 lb. per square foot. The allowable stresses are $f_c = 750$ lb. per square inch and $f_s = 16{,}000$ lb. per square inch. For these values $R = 133\frac{1}{3}$.

Since the bending moment, etc., depend partly upon the dead load a preliminary assumption, based on experience (or a second assumption based on trial and error) must be made for the thickness of the slab. The slab will be assumed to be 9 in. thick, and the effective depth $7\frac{1}{2}$ in.

$\therefore$ Effective span $= 14$ ft. $4\frac{1}{2}$ in. $+ 7\frac{1}{2}$ in. (since this is less than the wall thickness of 18 in.).
$= 15$ ft.

We deal with a strip of slab 1 ft. wide.

Dead load.

Asphalt wearing surface $= 10$
Slab $9 \times 12$ $= 108$

Total D.L. $= 118$ lb. per square foot.

Bending moment.

D.L.M. $\frac{1}{8} \times 118 \times 15^2 = 3{,}325$
L.L.M. $\frac{1}{8} \times 100 \times 15^2 = 2{,}815$

Total B.M. $= 6{,}140$ ft. lb.

Effective depth required (equation (29)).

$$d^2 = \frac{6{,}140}{133\frac{1}{3}} = 46 = (6\cdot8)^2.$$

Make the slab thickness $8\frac{1}{2}$ in. with $d = 7$ in. It is unnecessary to calculate the dead load moment again.

Methods will be given (1) without the use of design charts, using the formulæ on p. 40, and (2) using the design chart, *Fig.* 34A.

Method 1.

From equation (27) $A_s = \frac{M}{f_s jd}$. Care should be taken to keep the units the same in both the numerator and denominator, and so $M$ must be converted to in. lb. The value of $j$ may be assumed as 0·9.

Then
$$A_s = \frac{6{,}140 \times 12}{16{,}000 \times 0\cdot9 \times 7} = 0\cdot73 \text{ sq. in.}$$

The value of $f_c$ may be found by using equations (25) and (27).

Method 2.

$$R = \frac{6,140}{7^2} = 125\,;\quad p = 0{\cdot}0091\,;$$

$$A_s = 7 \times 12 \times 0{\cdot}0091 = 0{\cdot}76 \text{ sq. in.}\,;$$
$$f_c = 720 \text{ lb. per square inch.}$$

The small difference in value in the sectional area of steel obtained by the two methods is due to the assumption in Method 1 of a value for $j$ of 0·9, whereas it is more nearly 0·87 as shown by inspection of *Fig.* 34A.

The tensile reinforcement provided (see *Table* III) will be $\frac{3}{4}$-in. bars at 7-in. centres (the area of steel provided is 0·76 sq. in., and $\Sigma o = 4{\cdot}05$).

The cover below these bars will be made 1 in. so that the theoretical depth is $7\frac{1}{8}$ in.

## Shear.

The shear and bond stress in slabs of this type are generally negligible, and in the present case the experienced designer would not trouble to calculate them. As an example of method the calculations are given below for the section close to the support.

$$\text{D.L.V. } 7{\cdot}19 \times 118 = 848$$
$$\text{L.L.V. } 7{\cdot}19 \times 100 = 719$$

$$\text{Total } V = 1,567 \text{ lb.}$$

$$v = \frac{1,567}{12 \times 0{\cdot}9 \times 7} = 21 \text{ lb. per square inch.}$$

$$u = \frac{1,567}{4{\cdot}05 \times 0{\cdot}9 \times 7} = 61 \text{ lb. per square inch.}$$

It will be noted that $j$ is assumed as 0·9. The bending moment is small near the support, so that $R$ and $k$ are low and $j$ is high, and 0·9 is a safe assumption.

## Detailing the Slab.

*Fig.* 42 shows the detail drawing. The bond stress is reasonably low and no end hooks are required; all bars should, however, be run over the supports.

A slight camber should be provided in the transverse section for drainage, and in the longitudinal section for both drainage and appearance.

The transverse reinforcement, $\frac{1}{2}$-in. bars at 12-in. centres, shown in the drawing, are variously termed " wiring " bars, " shrinkage " or " temperature " bars, " distribution " bars. They fulfil all these functions. First, they assist in lining up the main bars and keeping them in position, as they are " wired " to them and help to form a mat. Secondly, they help to prevent cracks in the concrete from shrinkage or changes in temperature. (These are less likely to occur in a slab more or less free to contract and expand sideways than in a retaining wall or in a slab forming a panel surrounded by beams.) Thirdly, they assist in distributing in a transverse direction concentrated loads which may be brought on the slab.

Low kerbs may or may not be required at the sides of the slab. Drainage gullies may be required close to the walls.

The results of using richer concrete mixes may suitably be considered at this point.

(1) Design the same slab for an allowable concrete stress of 900 lb. per square inch, using the concrete mix of 120 lb. : 2 : 4 referred to in *Table* I, p. 38. The modular ratio is 15.

The higher allowable concrete stress results in the possibility of using a thinner slab. Assume a 7-in. slab. With the dead load at 94 lb. per square foot,

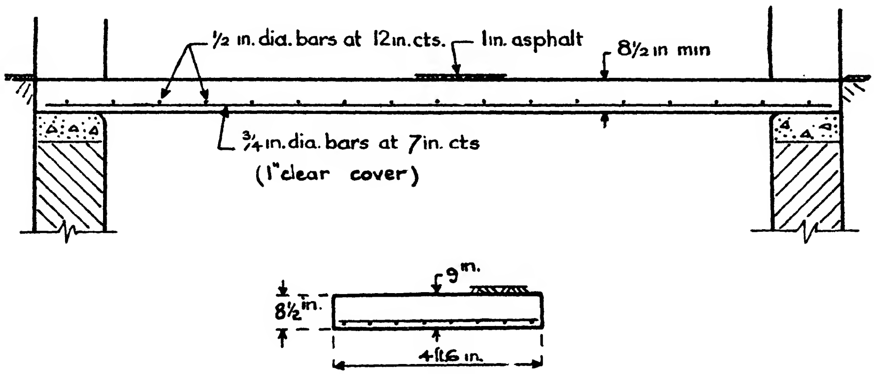



**Fig. 42.**

the total bending moment becomes 5,465 ft. lb. If we preserve 1 in. cover on the bars the effective depth (assuming $\frac{3}{4}$-in. bars) will be $5\frac{5}{8}$ in.

$$R = \frac{5{,}465}{(5\frac{5}{8})^2} = 172\tfrac{1}{2}\,; \quad p = 0{\cdot}0128\,;$$

$$f_c = 895 \text{ lb. per square inch.}$$

$$A_s = 5\tfrac{5}{8} \times 12 \times 0{\cdot}0128 = 0{\cdot}868 \text{ sq. in.}$$

Three-quarter-inch bars at 6-in. centres provide 0·88 sq. in.

(2) Taking the slab thickness and effective depth as in (1), but using a modular ratio of 12 (*Fig.* 34B)

$$R = 172\tfrac{1}{2}\,; \quad p = 0{\cdot}01255\,;$$

$$f_c = 955 \text{ lb. per square inch.}$$

These two cases should be compared with the original design. It is seen that the use of a stronger concrete results in a saving in that material and in the dead load, although owing to the decrease in effective depth a little extra steel is required.

An increase in the elastic modulus of the concrete, resulting in a decrease in the modular ratio, causes the concrete stress to rise while scarcely affecting the steel requirement.

# CHAPTER VI

## DESIGN OF CONTINUOUS SLABS

EXPERIENCE in design is of considerable help in many ways. It tells us when to depart from the theoretical calculations and modify detail by independent judgment. This cannot properly be taught in a textbook, and the beginner would do well to stick to the theory closely until such time as the wings of his experience will bear him with confidence. He can, however, even in the early stages, use his judgment in the application of theory so as to draw such distinctions as the one which now follows.

Continuous beams and slabs are generally designed by the Theorem of Three Moments, and the simple formula in equation (14) is most commonly used. This assumes a constant moment of inertia (rightly disregarding small variations of reinforcement and section), and unyielding supports. When the supports are rigid as in the case of columns supporting beams or walls supporting slabs, this assumption is reasonable so long as the foundations are good. When, however, lengthy beams support continuous slabs the deflection of the beams under live load provides a support not altogether rigid, and the formula is not truly applicable. In such a case it may be more reasonable to use a simple rule-of-thumb metnod such as first to find the " free " bending moment and then to introduce a reduction factor. The reduction factor to be used would depend upon the ratio of live to dead load. With approximately equal slab spans the dead load beam deflections are probably equal and the dead load bending moments are given accurately by the Theorem of Three Moments: bending moment coefficients are given in a number of textbooks; for a number of equal spans under uniform load the mid-span bending moment is one-third and the support moment two-thirds of the " free " span bending moment. With live load the moments at all sections are more severe. The mid-span moment is a maximum with alternate spans loaded (*Fig.* 28), and the support moment is a maximum with the two adjacent spans loaded and then alternate spans on either side (*Fig.* 29), and in both cases the live load bending moment is not very much less than the " free " span moment. When, therefore, a reduction factor is applied to the " free " span moment the ratio of dead to live load must be considered. When these are fairly comparable the bending moments at both mid-span and support are usually taken as two-thirds of the " free " moment (that is, $\frac{wl^2}{12}$ takes the place of $\left.\frac{wl^2}{8}\right)$. The end spans require special consideration because of the lack of end fixity. The positive moment in the end span with a

free end support may be taken as $\frac{wl^2}{10}$, and the same value should be used at the first interior support. With two spans only the central support moment is $\frac{wl^2}{8}$.

When slab spans are very unequal it is unsafe to use the reduction factor method, and the Theorem of Three Moments should be used.

EXAMPLE OF THE "REDUCTION FACTOR" METHOD.—A reinforced concrete floor slab is continuous for ten spans over long concrete beams each 18 in. wide and spaced at 12-ft. 6-in. centres, the slab and beams being monolithic. Design the slab (with a ½-in. granolithic wearing surface) to carry a live load of 100 lb. per square foot. The allowable value of $f_c$ is 750 lb. per square inch.

### Interior Spans.

The effective span is 11 ft. Assume that the total slab thickness is 5 in., with

$$d = 5 - 1 - \tfrac{1}{2} = 3\tfrac{1}{2} \text{ in.}$$

$$\text{D.L.} = 5 \times 12 = 60$$

$$\text{L.L.} = 100$$

$$\text{Total} = 160 \text{ lb. per square foot.}$$

B.M. (mid-span and support)

$$\frac{1}{12} \times 160 \times 11^2 = 1{,}600 \text{ ft. lb. approximately.}$$

(See *Fig.* 34A) $R = \frac{1{,}600}{(3\frac{1}{2})^2} = 131$ ; $p = 0{\cdot}0095$ ;

$$f_c = 740 \text{ lb. per square inch.}$$

$$A_s = 3\tfrac{1}{2} \times 12 \times 0{\cdot}0095 = 0{\cdot}40 \text{ sq. in.}$$

Use ½-in. bars at 6-in. centres, equivalent to 0·39 sq. in.

Generally it is scarcely economical to crank bars from the bottom to the top of the slab, but if this is done it is advisable to run alternate bars straight through

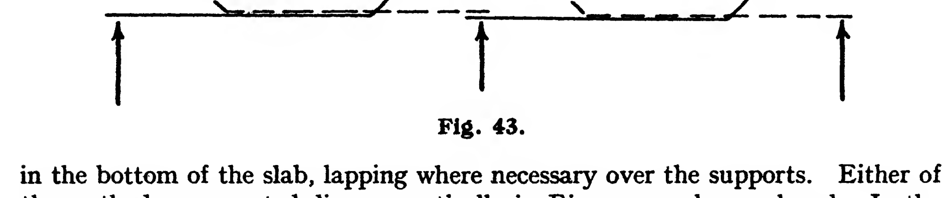


Fig. 43.

in the bottom of the slab, lapping where necessary over the supports. Either of the methods represented diagrammatically in *Fig.* 43 may be employed. In the

first, bars with two cranks are alternated with straight bars, while in the second, bars cranked as shown are provided and alternate bars are merely reversed. The points of inflexion (see p. 16) occur generally between the quarter-point and the support, but owing to the variability of the live load the position is not fixed; the top bars are made to run just beyond the quarter-point (it is sufficient if alternate bars do this) or "bond distance" beyond the support, whichever is the greater length.

A method which commends itself for simplicity, and which generally saves the cost of the slightly extra quantity of steel in the reduced cost of labour (bending and fixing) is that shown in *Fig.* 44, where the bars are run straight

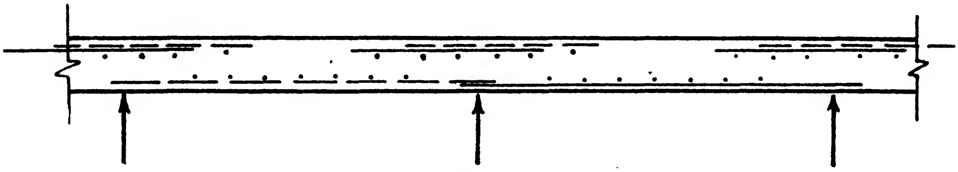


**Fig. 44.**

through in the bottom, and short straight bars are supplied in the top face over the supports; these top bars may be staggered to fit the bending moment diagram. Where the live load is appreciable compared with the dead load and negative moment can occur over the whole of a span a few top bars, say one in every three or four, should be run straight through. Distribution bars are supplied as indicated in *Fig.* 44.

Bond stresses are generally not calculated in slabs, as although these stresses are, in continuous slabs, often fairly high the bars have a fair length for anchorage, and it has been found in practice that bars provided for the bending moment requirements are safe also in bond: if there is slight local slip, failure is not likely to occur as a result of it. In the same way slab shear is frequently neglected. If it is calculated, and the moments have been determined by the reduction factor method, the shear is merely taken as one-half the slab-span load; the result will be slightly on the low side for live load. In the present example the calculation is made as follows:

$$V = 5\tfrac{1}{2} \times 160 \qquad = 880 \text{ lb.}$$

$$v = \frac{800}{12 \times 0{\cdot}9 \times 3\tfrac{1}{2}} = 23 \text{ lb. per square inch.}$$

## End Spans.

Where possible the end spans should be made a little shorter than the interior spans in order that the same slab section and reinforcement may be used. When this method is inconvenient the slab may have to be thickened as shown later.

The effective span should be taken to the centre of the outside support (owing to lack of rigidity),

$$\therefore\ l = 11 \text{ ft.} + 9 \text{ in.} = 11 \text{ ft. } 9 \text{ in.}$$

Assume a slab thickness (total) of 6 in., with $d = 6 - 1 - \tfrac{1}{2} = 4\tfrac{1}{2}$ in.

$$\text{D.L.} + \text{L.L.} = 172 \text{ lb. per square foot.}$$

B.M. (positive, and also negative at first interior support),

$$\frac{1}{10} \times 172 \times (11\tfrac{3}{4})^2 = 2,380 \text{ ft. lb.}$$

$$R = \frac{2,380}{(4\tfrac{1}{2})^2} = 117\,;\; p = 0\cdot0084$$

$$f_c = 690 \text{ lb. per square inch.}$$

$$A_s = 4\tfrac{1}{2} \times 12 \times 0\cdot0084 = 0\cdot455 \text{ sq. in.}$$

Use $\frac{1}{2}$-in. bars at 5-in. centres, equivalent to 0·47 sq. in.

The concrete stress is lower than the maximum allowable, but a $\frac{1}{2}$-in. reduction in slab thickness would bring the concrete stress above the allowable stress. It would, however, be reasonable to use a $5\frac{1}{2}$-in. slab if some compression reinforcement were supplied in the top of the slab.

Since the negative moment on the far side of the first interior support is equal to that on the near side, the effect of torsion in the beam being neglected, the concrete is slightly overstressed, but the condition is frequently neglected as the stress rapidly falls off away from the support section. In the example which follows it would be quite reasonable to make the slab only 6 in. thick (see p. 65), particularly in view of the fact that fillets would be used at the junction of beam and slab.

Example.—Design a continuous reinforced concrete slab for the arrangement of spans (effective) shown in *Fig.* 45. The supports consist of short stiff beams

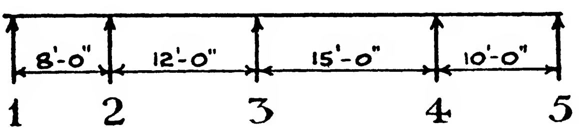



**Fig. 45.**

carried on columns, and may be taken as unyielding. A wearing surface will be laid weighing 8 lb. per square foot. The live load is to be taken as 100 lb. per square foot.

A rough idea of the floor thickness to be included for dead load may usually be got by providing for a bending moment say three-quarters of the "free" bending moment in the longest span. In the present example we assume a 7-in. thickness for the floor.

$$\text{Dead load} = (7 \times 12) + 8 = 92 \text{ lb. per square foot.}$$

Dead-load moments (from equation (14))

$$M_1 = 0 = M_5$$

$$8M_1 + 40M_2 + 12M_3 = -\tfrac{1}{4} \times 92(8^3 + 12^3)$$

$$\therefore \quad 40M_2 + 12M_3 = -51,520 \quad \ldots \quad (a)$$

$$12M_2 + 54M_3 + 15M_4 = -\tfrac{1}{4} \times 92(12^3 + 15^3)$$

$$= -117,369. \quad \ldots \quad (b)$$

$$15M_3 + 50M_4 + 10M_5 = -\tfrac{1}{4} \times 92(15^3 + 10^3)$$

$$\therefore \; 15M_3 + 50M_4 = -100,625 \quad \ldots \quad (c)$$

Eliminate $M_2$ between (*a*) and (*b*) and $M_4$ between (*c*) and (*d*),

$$(b) \times 10 \text{ reads } 120M_2 + 540M_3 + 150M_4 = -1{,}173{,}690 \quad \ldots \quad (b_1)$$
$$(a) \times 3 \text{ reads } 120M_2 + 36M_3 = -154{,}560 \quad \ldots \quad (a_1)$$
$$\text{Subtracting, } 504M_3 + 150M_4 = -1{,}019{,}130 \quad \ldots \quad (d)$$
$$(c) \times 3 \text{ gives } 45M_3 + 150M_4 = -301{,}875$$
$$\text{Subtracting, } 459M_3 = -717{,}255$$
$$\therefore M_3 = -1{,}561 \text{ ft. lb.}$$

Substituting this value in equation (*c*)

$$-23{,}415 + 50M_4 = -100{,}625$$
$$\therefore M_4 = -\frac{77{,}210}{50} = -1{,}544 \text{ ft. lb.}$$

Substituting for $M_3$ in equation (*a*)

$$40M_2 - 18{,}732 = -51{,}520$$
$$\therefore M_2 = -\frac{32{,}788}{40} = -820 \text{ ft. lb.}$$

Each span is loaded in turn, and the moments at each support calculated. Special care should be taken with the algebraic signs as some final moments will be positive.

Live load on span 1–2

$$0 + 40M_2 + 12M_3 = -\tfrac{1}{4} \times 100 \times 8^3$$
$$= -12{,}800 \quad \ldots \quad (e)$$
$$12M_2 + 54M_3 + 15M_4 = 0 \quad \ldots \quad (f)$$
$$15M_3 + 50M_4 + 0 = 0 \quad \ldots \quad (g)$$

Eliminate $M_4$ between (*f*) and (*g*)

$$(f) \times 2 \text{ gives } 24M_2 + 108M_3 + 30M_4 = 0$$
$$(g) \times \tfrac{3}{5} \text{ gives } 9M_3 + 30M_4 = 0$$
$$\text{Subtracting, } 24M_2 + 99M_3 = 0 \quad \ldots \quad (h)$$

Eliminate $M_2$ between (*h*) and (*e*)

$$(h) \times \tfrac{5}{3} \text{ reads } 40M_2 + 165M_3 = 0$$
$$(e) \quad 40M_2 + 12M_3 = -12{,}800$$
$$\text{Subtracting, } 153M_3 = +12{,}800$$
$$\therefore M_3 = +83{\cdot}7 \text{ ft. lb.}$$

Substituting this value in equation (*e*)

$$40M_2 + 1{,}005 = -12{,}800$$
$$M_2 = -\frac{13{,}805}{40} = -345 \text{ ft. lb.}$$

Substituting for $M_3$ in equation (*g*)

$$1{,}256 + 50M_4 = 0$$
$$\therefore M_4 = -\frac{1{,}256}{50} = -25 \text{ ft. lb.}$$

Live load on span 2–3

$$0 + 40M_2 + 12M_3 = -\tfrac{1}{4} \times 100 \times 12^3$$
$$= -43{,}200 \quad \ldots \quad (i)$$
$$12M_2 + 54M_3 + 15M_4 = -43{,}200 \quad \ldots \quad (j)$$
$$15M_3 + 50M_4 + 0 = 0 \quad \ldots \quad (k)$$

Eliminate $M_4$ between $(j)$ and $(k)$

$(j) \times 2$ gives $24M_2 + 108M_3 + 30M_4 = -86{,}400$

$(k) \times \frac{3}{5}$ gives $9M_3 + 30M_4 = 0$

Subtracting, $24M_2 + 99M_3 = -86{,}400 \quad \ldots \quad (l)$

Eliminate $M_2$ between $(l)$ and $(i)$

$(l) \times \frac{5}{3}$ reads $40M_2 + 165M_3 = -144{,}000$

$(i)$ $40M_2 + 12M_3 = -43{,}200$

Subtracting, $153M_3 = -100{,}800$

$\therefore M_3 = -659$ ft. lb.

Substituting this value in equation $(i)$

$$40M_2 - 7{,}908 = -43{,}200$$

$$\therefore M_2 = -\frac{35{,}292}{40} = -882 \text{ ft. lb.}$$

Substituting for $M_3$ in equation $(k)$

$$-9{,}885 + 50M_4 = 0$$

$$\therefore M_4 = +\frac{9{,}885}{50} = +198 \text{ ft. lb.}$$

Live load on span 3–4

$$0 + 40M_2 + 12M_3 = 0 \quad \ldots \quad (m)$$
$$12M_2 + 54M_3 + 15M_4 = -\tfrac{1}{4} \times 100 \times 15^3$$
$$= -84{,}375 \quad \ldots \quad (n)$$
$$15M_3 + 50M_4 + 0 = -84{,}375 \quad \ldots \quad (o)$$

Eliminate $M_4$ between $(n)$ and $(o)$

$(n) \times 2$ gives $24M_2 + 108M_3 + 30M_4 = -168{,}750$

$(o) \times \frac{3}{5}$ gives $9M_3 + 30M_4 = -50{,}625$

Subtracting, $24M_2 + 99M_3 = -118{,}125. \quad \ldots \quad (p)$

Eliminate $M_2$ between $(p)$ and $(m)$

$(p) \times \frac{5}{3}$ gives $40M_2 + 165M_3 = -196{,}875$

$(m)$ $40M_2 + 12M_3 = 0$

Subtracting, $153M_3 = -196{,}875$

$\therefore M_3 = -1{,}287$ ft. lb.

Substituting this value in equation $(m)$

$$40M_2 - 15{,}444 = 0$$

$$\therefore M_2 = +\frac{15{,}444}{40} = +386 \text{ ft. lb.}$$

Substituting for $M_3$ in equation $(o)$

$$-19{,}305 + 50M_4 = -84{,}375$$

$$\therefore M_4 = -\frac{65{,}070}{50} = -1{,}301 \text{ ft. lb.}$$

Live load on span 4–5

$$0 + 40M_2 + 12M_3 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (q)$$

$$12M_2 + 54M_3 + 15M_4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (r)$$

$$15M_3 + 50M_4 + 0 = -25{,}000 \quad . \quad . \quad . \quad . \quad . \quad (s)$$

Eliminate $M_4$ between ($r$) and ($s$)

($r$) $\times$ 2 gives $24M_2 + 108M_3 + 30M_4 = 0$

($s$) $\times \frac{3}{5}$ gives $9M_3 + 30M_4 = -15{,}000$

Subtracting, $24M_2 + 99M_2 = +15{,}000$ . . . . . ($t$)

Eliminate $M_2$ between ($t$) and ($q$)

($t$) $\times \frac{5}{3}$ gives $40M_2 + 165M_3 = +25{,}000$

($q$) $40M_2 + 12M_3 = 0$

Subtracting, $153M_3 = +25{,}000$

$\therefore M_3 = +164$ ft. lb.

Substituting this value in equation ($q$)

$$40M_2 + 1{,}962 = 0$$

$$\therefore M_2 = -\frac{1{,}962}{40} = -49 \text{ ft. lb.}$$

Substituting for $M_3$ in equation ($s$)

$$2{,}452 + 50M_4 = -25{,}000$$

$$\therefore M_4 = -\frac{27{,}452}{50} = -549 \text{ ft. lb.}$$

These moments should now be tabulated so that the maximum combinations can be found. The dead load moments must all be included, and those live loads contributing a moment of similar algebraic sign for the section under consideration should also be included.

In the design of a complete structure the shears should be obtained also in the manner shown on p. 23, and these also would be tabulated in order to obtain the maximum possible load per foot run on the beams which support the slab. The loading in *Fig.* 29 would give the maximum load on the beam forming the support between the two loaded spans.

Summary of Bending Moments (ft. lb.).

| Loading {D.L. 92 lb. per square foot. L.L. 100 lb. per square foot. | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|
| Dead Load . . . . . . . . . . . . . | − 820 | − 1,561 | − 1,544 |
| L.L. on Span 1–2 . . . . . . . . . . | − 345 | + 84 | − 25 |
| L.L. on Span 2–3 . . . . . . . . . . | − 882 | − 659 | + 198 |
| L.L. on Span 3–4 . . . . . . . . . . | + 386 | − 1,287 | − 1,301 |
| L.L. on Span 4–5 . . . . . . . . . . | − 49 | + 164 | − 549 |
| Combined Loadings . . . . . . . . . | − 2,096 | − 3,507 | − 3,419 |

In order to obtain positive moments it will be necessary to tabulate the "free" moments ($= \frac{1}{8}wl^2$) in each span.

"FREE" BENDING MOMENTS (ft. lb.).

| Loading. | Span 1–2, 8 ft. | Span 2–3, 12 ft. | Span 3–4 15 ft. | Span 4–5, 10 ft. |
|---|---|---|---|---|
| Dead Load, 92 lb. per square foot . | 736 | 1,656 | 2,588 | 1,150 |
| Live Load, 100 lb. per square foot . | 800 | 1,800 | 2,812 | 1,250 |

The maximum bending moments at the support and interior of the span will be designed for in this example. In most cases the same slab thickness would be used throughout and the reinforcing steel would be varied from span to span in as simple a manner as possible, employing perhaps a number of bars to run straight through two spans. In exceptional cases the floor thickness might be varied. The student can, as an exercise, calculate the steel required in the remaining spans.

MAXIMUM MOMENT AT THE SUPPORT.—This occurs at support No. 3 with spans 2–3 and 3–4 loaded, and equals 3,507 ft. lb. To find the slab thickness required

$$d^2 = \frac{3{,}507}{133\frac{1}{2}} = (5{\cdot}13)^2. \quad \text{Make } d = 5\tfrac{1}{2} \text{ in.}$$

Make floor $6\frac{1}{2}$ in. thick.*

$$R = \frac{3{,}507}{(5\frac{1}{2})^2} = 116. \qquad p = 0{\cdot}0084.$$

$$f_c = 685 \text{ lb. per square inch.}$$

$$A_s = 5\tfrac{1}{2} \times 12 \times 0{\cdot}0084 = 0{\cdot}555 \text{ sq. in.}$$

Five-eighth-inch bars at 6-in. centres are equivalent to 0·61 sq. in.

MAXIMUM POSITIVE MOMENT.—This occurs in span 3–4 when spans 1–2 and 3–4 are loaded. The "free" bending moment (D.L. and L.L. combined) is drawn for this span, its shape being a parabola; the maximum ordinate is $\frac{1}{8} \times 192 \times 15^2$ = 5,400 ft. lb. The support moments at 3 and 4 for the condition of loading

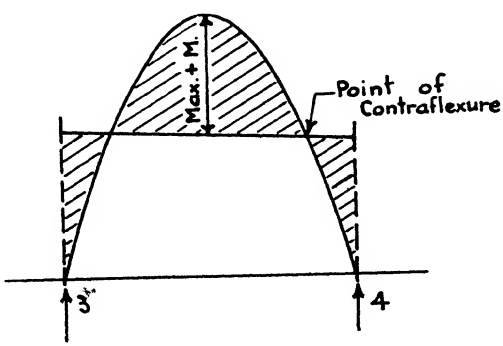



Fig. 46.

* Floor thickness was assumed to be 7 in., and there is no need to revise dead load moments. In cases where the error is appreciable the dead load moments are revised in direct proportion to the dead load. In the present example the floor thickness could if desired be reduced to 6 in.

already referred to are scaled up over the supports and the points so obtained are joined by a straight line. The resulting B.M.D. is shown in *Fig.* 46.

(D.L. + L.L.) moment at support 3 = − 1,561 + 84 − 1,287 = − 2,764 ft. lb.

" " " " 4 = − 1,544 − 25 − 1,301 = − 2,870 ft. lb.

Another method, obvious to those familiar with the calculus, is to find the point in the span where the shear is zero; the maximum positive moment occurs at this section. This method is demonstrated in the following calculations.

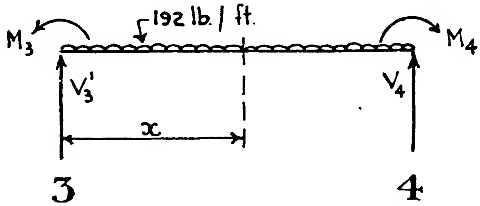



**Fig. 47.**

SECTION WHERE SHEAR IS ZERO (see *Fig.* 47).—To find $V_3'$ take moments about support No. 4 (clockwise moments positive). Because of equilibrium

$$M_4 - M_3 + V_3'l - \frac{wl^2}{2} = 0$$

$$2{,}870 - 2{,}764 + 15V_3' - \frac{192 \times 15^2}{2} = 0$$

$$\therefore \; V_3' = \frac{21{,}600 + 2{,}764 - 2{,}870}{15}$$

$$= 1{,}433 \text{ lb.}$$

Section of zero shear is distant $x$ ft. from support No. 3.

Then
$$1{,}433 - 192x = 0$$

$$\therefore \; x = \frac{1{,}433}{192} = 7{\cdot}46 \text{ ft.}$$

This point is practically at mid-span, but with different arrangements of span and loading the maximum moment could occur farther away.

The moment at $x$ is

$$M_x = -2{,}764 + 1{,}433 \times 7{\cdot}46 - \frac{192 \times (7{\cdot}46)^2}{2}$$

$$= -2{,}764 + 10{,}700 + 5{,}340 = 2{,}596 \text{ ft. lb.}$$

$$R = \frac{2{,}596}{(5\tfrac{1}{2})^2} = 86 \qquad p = 0{\cdot}00605$$

$$f_c = 560 \text{ lb. per square inch.}$$

$$A_s = 5\tfrac{1}{2} \times 12 \times 0{\cdot}0061 = 0{\cdot}40 \text{ sq. in.}$$

Half-inch bars at 6-in. centres are equivalent to 0·39 sq. in.

The detailing would be done in a manner similar to that shown in *Fig.* 44.

When the spans are equal and the live load includes concentrations it is advisable to use standard influence lines such as are given in numbers of text-books. *Figs.* 31, 31A, 31B, or 31C will suffice for practically all cases.

Whenever a structure is symmetrical (see p. 18) in form and loading this fact should be made use of as we can tell at once that the forces also are symmetrical—thus in *Fig.* 48 we can write down the following statements from inspection, and they will reduce the work of calculation to a minimum.

$$M_1 = M_6,\ M_2 = M_5,\ M_3 = M_4.$$

Similarly if span 2–3 (*Fig.* 48), say, alone carries live load, bending moments

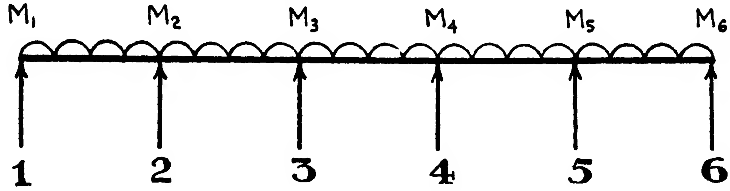



Fig. 48.

$M_4$ and $M_5$ are equal respectively to $M_3$ and $M_2$ for the condition when span 4–5 alone is loaded. The principle of symmetry should be exploited (with care) to its utmost.

# CHAPTER VII

## DESIGN OF SIMPLY-SUPPORTED BEAMS

THERE is little difference in method between the design of slabs and beams, the difference arises mainly in the detailing. The conditions of loading frequently differ as slabs more generally carry a distributed load while a beam is often called upon to support moving concentrated loads. When a series of concentrated loads moves over a span, fixed intervals being preserved between loads, the maximum live load moment occurs in the span when the centre of gravity of the system (that is, all loads on the span) and the unit nearest to it equally straddle the mid-span point, and the section at which the maximum live load moment occurs is immediately below the unit referred to.* Thus in *Fig.* 49 the centre of

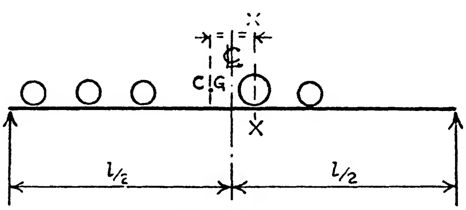



**Fig. 49.**

gravity of the system and the concentration at the section $XX$ are equidistant from the middle of the span, and the maximum bending moment occurs at $XX$. Where the concentrations vary in magnitude the nearest unit will usually be the largest, as the centre of gravity naturally tends towards this position.

A beam is like a narrow strip of slab deepened so that the overall depth is generally from $1\frac{1}{4}$ to $2\frac{1}{2}$ times the breadth. The depth is necessary on account of the large bending moments and shears resulting from heavier loading than is encountered in slabs. Beams are often used to support slabs, and they are then almost invariably incorporated with the slab in monolithic construction and designed as T-beams as will be described later. At the moment we are concerned only with the design of the simple rectangular beam which has no slab to stiffen it sideways. The top portion of the beam is in compression and if the breadth of the beam is small compared with the span there is a tendency for the top of the beam to buckle in the same way that a steel joist with a very narrow compression flange will sometimes do. Such failures in concrete beams are very uncommon, but the breadth $b$ should not be less than $\frac{1}{25} \times$ the span.

* This general statement is easily proved with the aid of the calculus.

Shear stresses should always be calculated in beams in the manner described on pp. 47 and 56, and web reinforcement should be carefully detailed. The maximum bending moments can be found at the various sections by moving the live load concentrations along; in addition to mid-span they should be calculated at sections where it is proposed to bend up bars from the main group in order to provide shear resistance for the web.

The first example will be a simple one with low shear stresses, and a subsequent example will show why T-beams are used and the effect such use has on the shear stresses.

EXAMPLE.—Design a simple reinforced concrete beam to carry a rolling load of 6 tons from a hoist over a clear span of 20 ft. Allow for an impact factor of $\frac{2}{3}$.

Allowable stresses, $f_c = 800$ lb. per square inch, $f_s = 16{,}000$ lb. per square inch.

A suitable section (obtained by trial and error) is shown in *Fig.* 50.

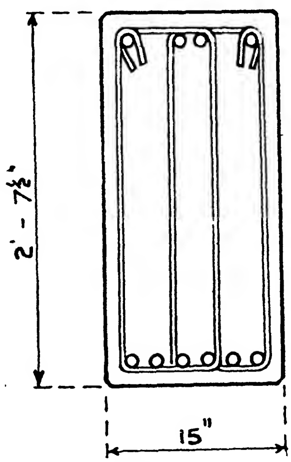



**Fig. 50.**

The effective span may be taken as $20 + d = 22\frac{1}{2}$ ft. say.

Dead load $= 15 \times 31\frac{1}{2} = 472$ lb. per foot run.

Live load $= 6 \times 1\frac{2}{3} \times 2{,}240 = 22{,}400$ lb. concentrated load.

The maximum B.M. occurs at mid-span,

$$\begin{aligned} \text{D.L.}\ \tfrac{1}{8} \times 472 \times 22\tfrac{1}{2}^2 &= 30{,}000 \\ \text{L.L.}\ \tfrac{1}{4} \times 22{,}400 \times 22\tfrac{1}{2} &= 126{,}000 \\ \hline & \quad 156{,}000 \text{ ft. lb.} \end{aligned}$$

The design may be made by either of the methods demonstrated in Chapter V. The second method employing *Fig.* 34A will be used here.

$$R = \frac{M}{bd^2} = \frac{156{,}000}{1\frac{1}{4} \times 29^2} = 148$$

$$p = 0{\cdot}0108\ ;\ f_c = 800 \text{ lb. per square inch.}$$

$$A_s = 15 \times 29 \times 0{\cdot}0108 = 4{\cdot}7 \text{ sq. in.}$$

Six 1-in. bars have a cross-sectional area of 4·71 sq. in.

The maximum shear occurs at the support, with the load at this section.

$$V = (10 \times 472) + 22{,}400 = 27{,}120 \text{ lb.}$$

$$v = \frac{27{,}120}{15 \times 0{\cdot}9 \times 29} = 69 \text{ lb. per square inch.}$$

This is low, and theoretically stirrups are not required. In practice stirrups would be provided to act in the capacity of " temperature " steel and to support the tensile reinforcement. In the condition where the live load is applied to the underside of the beam these bars would act as " hanging " steel to transfer the applied load to the top of the beam, and such steel would be supplied in addition to any shear reinforcement which might be required. In the present example $\frac{1}{2}$-in. diameter double links will be provided at 15-in. centres $\left(\frac{d}{2}\right)$, and these will be suspended from four light bars placed in their top corners. Supposing the live load to be hung from a rail bolted into the bottom of the beam, it may reasonably be assumed that eight arms of the links will assist in supporting the load in any position: the direct tension in these bars acting as hangers is $\frac{22{,}400}{8 \times 0{\cdot}196} = 14{,}300$ lb. per square inch.

As the shear stress is low it will be unnecessary to bend up any of the bottom bars. In practice many designers would bend up one pair, as it has been found that this adds to the strength of the beam, although a single pair of bent-up bars should not be relied upon as shear reinforcement. The bars if bent up should be hooked over the supports.

Assuming two bars to be bent up, four bars would be carried in the bottom of the beam over the supports. These provide a periphery ($\Sigma o$) of 12·56 in. (see *Table* III, p. 45). The bond stress is therefore

$$u = \frac{27{,}120}{12{\cdot}56 \times 0{\cdot}9 \times 29} = 83 \text{ lb. per square inch.}$$

This is reasonable, but it is good practice in all beams to hook the ends of the bars for anchorage over the supports, and this will be done in the present case.

## T-Beams.

Equation (27) showed that the strength of a reinforced concrete beam is controlled by one or both of two stress factors, $f_s$ and $f_c$. When the design is perfectly balanced both stresses are a maximum together. In the example just given it would have been possible to increase the steel to meet the requirements of any additional load, but this would have left the concrete overstressed in compression. There are two ways of meeting this deficiency: one is to increase the concrete area by deepening or broadening the beam, and the other is to provide compression reinforcement. The latter method will be discussed in the next chapter; the former will be considered now. To deepen the beam beyond a certain stage becomes undesirable from the point of view either of headroom or economy, so we consider broadening the beam. Let us glance again for a moment at *Fig.* 33 on p. 40. It is seen that only the top portion of the beam is in compression so that, apart from considerations of shear, it is necessary to widen only

this portion. The width of beam necessary may thus be obtained from formula (28) which may be rewritten $b = \frac{M}{Rd^2}$ and the portion of beam below the neutral axis may be considerably reduced in breadth so long as the shear stresses are adequately provided for and the tensile reinforcement can be accommodated. It may be further noticed from *Fig.* 33 that the really efficient sectional area of concrete is the top portion near the extreme fibres ; this is owing to the fact that it is most highly stressed and also has most leverage contributing towards the resisting moment. It is therefore economical to widen the flange (as we call it) still further and to keep the underside above the neutral axis, so increasing the effective depth. The effect is shown in *Fig.* 51, and it will be seen that the saving

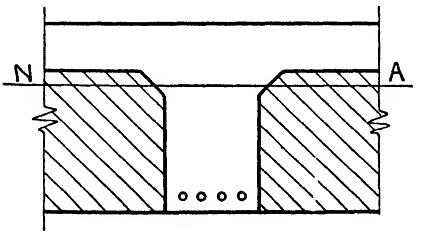



**Fig. 51.**

in concrete by eliminating the shaded portions considerably reduces the dead load.

In many structures the flange is ready to hand. A deck slab is provided to carry floor loads, and this slab is stressed in a direction at right angles to the beams which support it : if the slab is constructed monolithic with the beams it acts as the flange of the T-beam and becomes stressed in a direction at right angles to the first stresses. Such double usage of the slab is legitimate and efficient. The thickness of the flange is thus decided by the design of the floor slab. It is unwise, however, to combine very thin slabs with very heavy beams, a reasonable relationship should be maintained.

When beams are widely spaced the full slab width is not considered in the T-beam. The width of slab assumed for flange area is generally limited to the smallest of the three following dimensions :

(1) From mid-span to mid-span of deck slab,
(2) Twelve times the slab thickness, and
(3) One-quarter of the effective span of the T-beam.

Equations (23) and (24) hold as in the rectangular beam,* and $f_s$ is obtained from equation (27) rewritten in this form,

$$f_s = \frac{M}{A_s jd} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27\text{A})$$

---

* $$f_c = \frac{f_s k}{n(1 - k)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The condition is shown in *Fig.* 52. The compression in the stem between the neutral axis and the underside of the slab is neglected, and by consideration

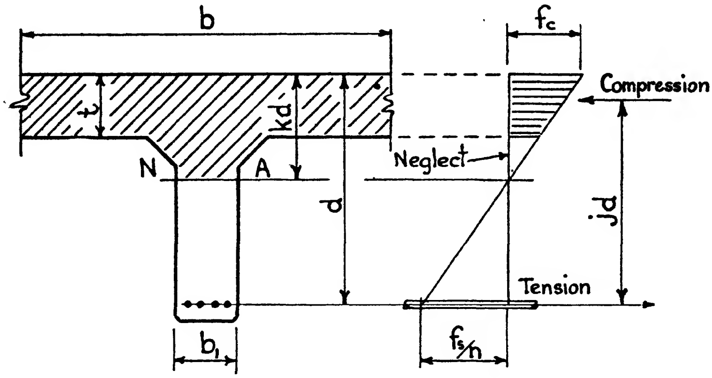



**Fig. 52.**

of similar triangles, and then equating total compression to total tension, we derive the following formulæ.

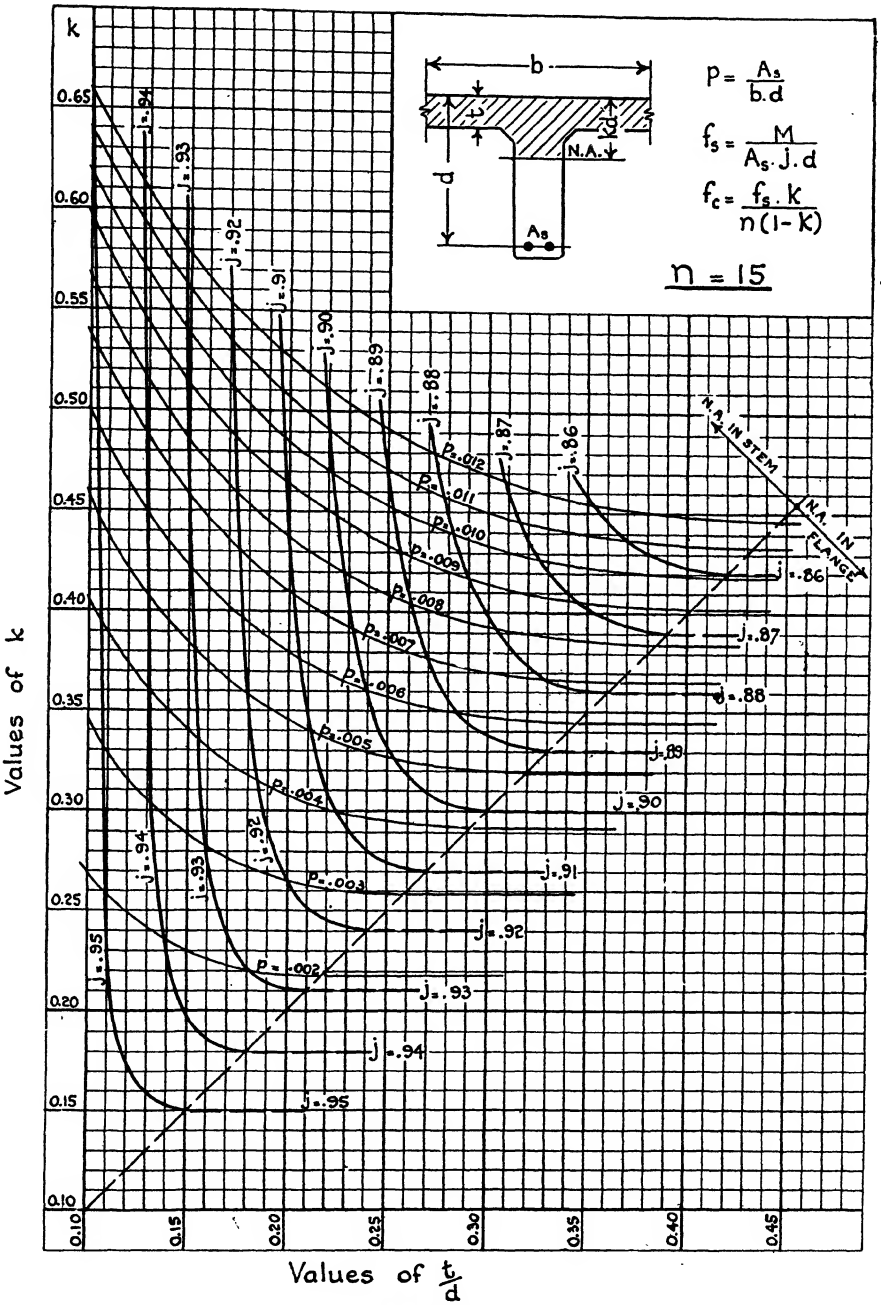
$$k = \frac{\left(\frac{t}{d}\right)^2 + 2pn}{2\left(\frac{t}{d} + pn\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

$$j = \frac{\left(\frac{t}{d}\right)^3 + 4\left(\frac{t}{d}\right)^2 pn - 12\frac{t}{d}pn + 12pn}{6pn\left(2 - \frac{t}{d}\right)} \quad . \quad . \quad . \quad . \quad (37)$$

It would be laborious and unnecessary to calculate these each time, and curves have been plotted from these equations and are given in *Figs.* 53A, 53B, and 53C for modular ratios of 15, 12, and 10. If the neutral axis falls inside the flange the values of $k$ and $j$ may be read from the straight lines indicated for this condition, or alternatively the ordinary beam formulæ could be used.

The steel ratio $p$ is calculated on the flange width $b$, and $d$. Owing to the limited area of stem available for placing the tensile reinforcement it is often necessary to provide more than one layer of bars; $d$ is then taken to the centre of gravity of the group, and the stress in the various layers of bars will be in direct proportion to their distances from the neutral axis. In shallow beams the upper layers will be working inefficiently. Bars in upper layers should lie vertically above bars in the lower layers (not staggered) and plenty of space for concrete should be provided between all bars.

Horizontal shear may be found to be high between flange and stem, and both units should be poured together. Bent-up bars and stirrups should extend well up into the slab. Deck reinforcement will provide transverse bars necessary for the proper functioning of the flange, and it is advisable to detail fillets at the junctions of beams and slabs.



k
Values of k
Values of $\frac{t}{d}$
b
t
kd
N.A.
d
As
$p = \frac{A_s}{b.d}$
$f_s = \frac{M}{A_s.j.d}$
$f_c = \frac{f_s.k}{n(1-k)}$
n = 15
N.A. IN STEM
N.A. IN FLANGE
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
j=.95
j=.94
j=.93
j=.92
j=.91
j=.90
j=.89
j=.88
j=.87
j=.86
p=.002
p=.003
p=.004
p=.005
p=.006
p=.007
p=.008
p=.009
p=.010
p=.011
p=.012


Fig. 53A.

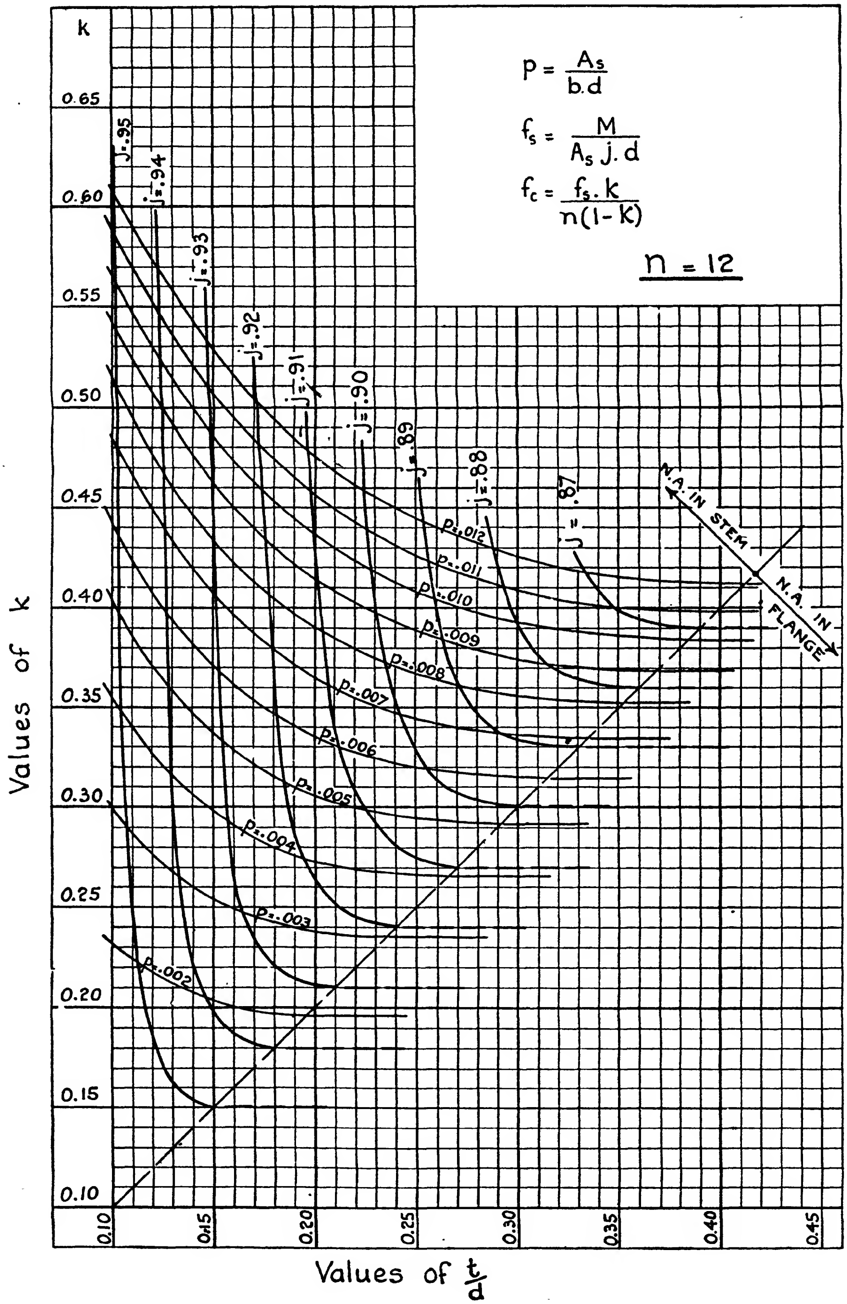

k
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
Values of k
Values of t/d
p = As / b.d
fs = M / As j. d
fc = fs. k / n(1-k)
n = 12
j=.95
j=.94
j=.93
j=.92
j=.91
j=.90
j=.89
j=.88
j=.87
p=.012
p=.011
p=.010
p=.009
p=.008
p=.007
p=.006
p=.005
p=.004
p=.003
p=.002
N.A. IN STEM
N.A. IN FLANGE

**Fig. 53B.**

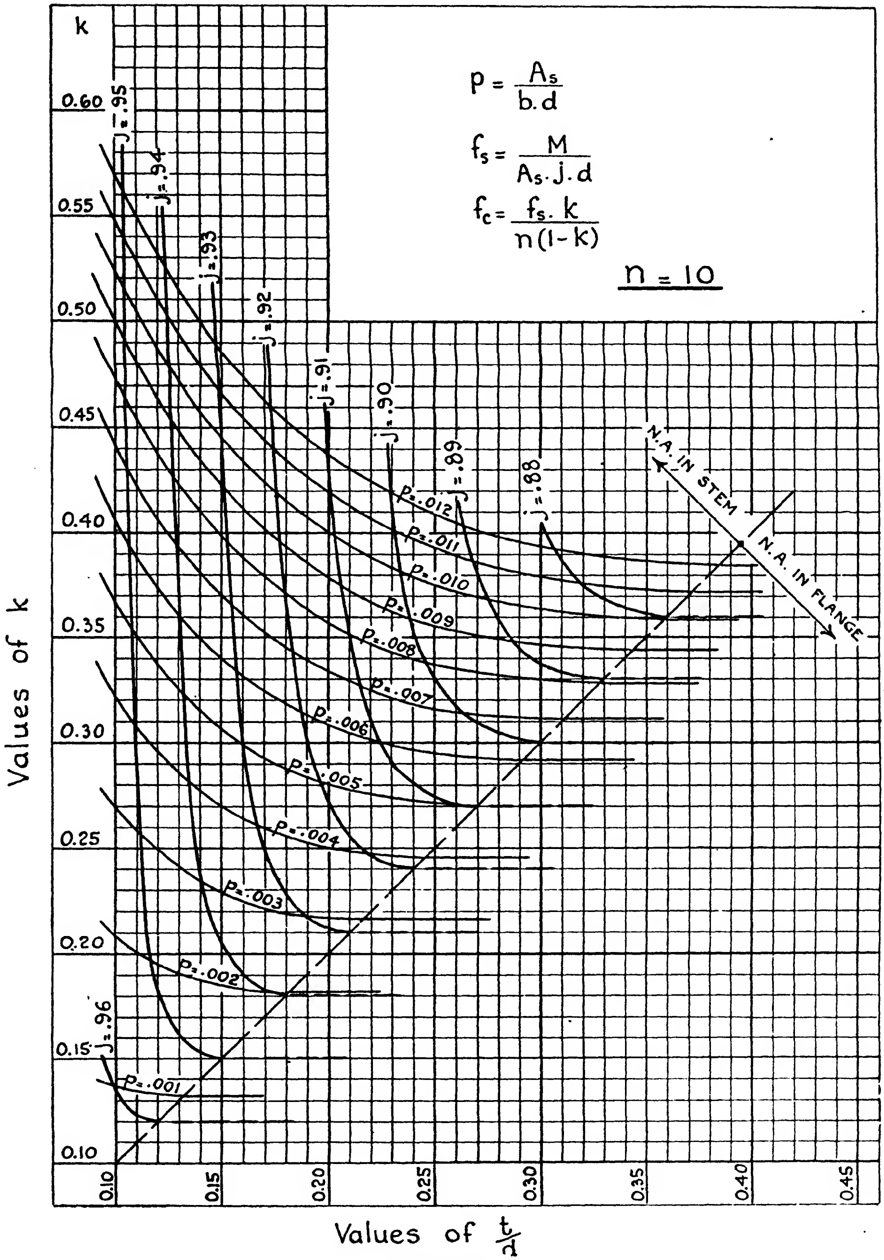

k
$p = \frac{A_s}{b.d}$
$f_s = \frac{M}{A_s . j . d}$
$f_c = \frac{f_s . k}{n(1-k)}$
n = 10
0.60
0.55
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
j = .95
j = .94
j = .93
j = .92
j = .91
j = .90
j = .89
j = .88
j = .96
p = .012
p = .011
p = .010
p = .009
p = .008
p = .007
p = .006
p = .005
p = .004
p = .003
p = .002
p = .001
N.A. IN STEM
N.A. IN FLANGE
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
Values of k
Values of t/d

Fig. 53c.

By referring to *Fig.* 35 on p. 46 it will be seen that the unit shear in the flange is low, and it has been found experimentally that the resistance of a T-beam to shear (diagonal tension) is about equal to that of a rectangular beam having the same width of section as the stem; the same formula * is therefore used, $b$ being taken as the width of the stem. Web reinforcement consisting of bent-up bars and stirrups is provided exactly as for rectangular beams (see p. 47).

EXAMPLE.—The rolling load in the previous example is increased so that with impact included it becomes 30,000 lb. This load travels along the top of the beam which is monolithic with a 6-in. concrete floor slab spanning to the adjacent beams which are 8 ft. away on either side. When the concentrated load is present live load on the slab may be disregarded. Design the beam.

The ratio of effective depth of beam to span in the previous example was $\frac{2{\cdot}42}{22{\cdot}5} = \frac{1}{10}$ approx. This is not unreasonable and will be repeated. A width of 12 in. may be tried.

Dead load per ft. run of beam.

$$\begin{aligned} \text{From deck slab } 8 \times \tfrac{1}{2} \times 150 &= 600 \\ \text{From beam (assumed) } 2{\cdot}125 \times 1 \times 150 &= 320 \\ \text{Fillets, say, } &= \phantom{0}30 \\ &\overline{950 \text{ lb.}} \end{aligned}$$

Check section for shear.

$$V = (10 \times 950) + 30{,}000 = 39{,}500 \text{ lb.}$$

$$v = \frac{39{,}500}{12 \times 0{\cdot}9 \times 29} = 126 \text{ lb. per square inch.}$$

This is satisfactory, and unless there is any special reason for reducing the beam width below 12 in. this dimension will be retained. The proportions are satisfactory, and there is convenient space for the provision of reinforcing bars.

The flange width considered in design is the smallest of the following (see p. 71).

(1) 8 ft. $= 96$ in.
(2) $12 \times 6 = 72$ in.
(3) $\frac{1}{4} \times 22\frac{1}{2} \times 12 = 67\frac{1}{2}$ in.

The section is shown in *Fig.* 54.

Bending moment at mid-span

$$\begin{aligned} \text{D.L. } \tfrac{1}{8} \times 950 \times 22\tfrac{1}{2}^2 &= \phantom{0}60{,}200 \\ \text{L.L. } \tfrac{1}{4} \times 30{,}000 \times 22\tfrac{1}{2} &= 168{,}800 \\ &\overline{229{,}000 \text{ ft. lb.}} \end{aligned}$$

Approximate amount of steel required† $= \dfrac{229{,}000}{16{,}000 \times 2{\cdot}08} = 6{\cdot}86$ sq. in.

* Equation (30), $v = \dfrac{V}{bjd}$.

† The lever arm $jd$ is assumed as follows: two layers of bars will probably be required in the tension group, and the centre of compression will be between one-third and one-half of the flange thickness from the top of the beam. These considerations suggest a dimension of 25 in. If $M$ is kept in ft. lb. units $d$ should be taken in feet.

$$\text{Try}\begin{cases}\text{Five 1-in. bars} = 3{\cdot}927 \text{ (in lower layer for efficiency)}\\ \text{Five } \tfrac{7}{8}\text{-in. bars} = 3{\cdot}006\end{cases}$$

$$\overline{6{\cdot}93 \text{ sq. in.}}$$

Distance of centre of gravity of the group from the bottom face (see *Fig.* 54).

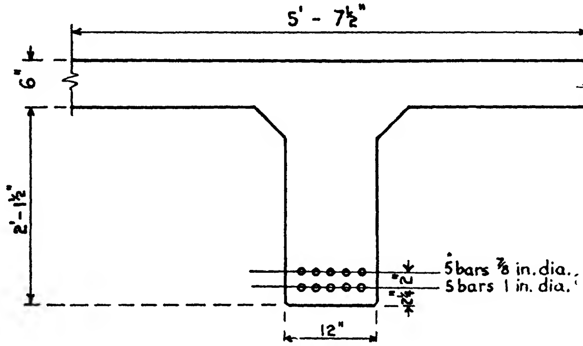



**Fig. 54.**

$$\begin{aligned} 3{\cdot}93 \times 2\tfrac{1}{4} &= 8{\cdot}85 \\ 3{\cdot}00 \times 4\tfrac{1}{4} &= 12{\cdot}75 \\ \hline & \quad 21{\cdot}60 \end{aligned}$$

$$\frac{21{\cdot}60}{6{\cdot}93} = 3{\cdot}1 \text{ in.} \quad \therefore\ d = 31{\cdot}5 - 3{\cdot}1 = 28{\cdot}4 \text{ in.}$$

$$\text{Steel ratio } p = \frac{6{\cdot}93}{67\frac{1}{2} \times 28{\cdot}4} = 0{\cdot}00362$$

$$\frac{t}{d} = \frac{6}{28{\cdot}4} = 0{\cdot}211.$$

From *Fig.* 53A we read for these values, $j = 0{\cdot}915$ and $k = 0{\cdot}287$, and the average steel stress is

$$f_s = \frac{229{,}000 \times 12}{6{\cdot}93 \times 0{\cdot}915 \times 28{\cdot}4} = 15{,}280 \text{ lb. per square inch.}$$

The neutral axis is $0{\cdot}287 \times 28{\cdot}4 = 8{\cdot}15$ in. from the top face. The distance from the neutral axis to the centre of gravity of the steel is $28{\cdot}4 - 8{\cdot}15 = 20{\cdot}25$ in., and the distance to the lower layer of bars is 21·1 in.

$$\therefore\ f_s \text{ (in the lower bars)} = \frac{15{,}280 \times 21{\cdot}1}{20{\cdot}25} = 15{,}900 \text{ lb. per square inch.}$$

$$f_c = \frac{15{,}280 \times 0{\cdot}287}{15 \times 0{\cdot}713} = 410 \text{ lb. per square inch.}$$

The beam should now be set out in elevation on the drawing-board and the top layer of bars bent up as shown in *Fig.* 55. The pair of outer bars is bent up at, say, 30 deg. so as to hook conveniently over the support as shown. The next

pair is then bent up a distance of $\frac{3}{4}d$ or approximately 21 in. measured along the neutral axis, and the third bar at a similar distance farther along.

The steel area available should be checked against the requirement at the bends. The single bar makes only a small reduction, and one check at the section where the pair of bars is bent up about 5 ft. from mid-span should suffice in the

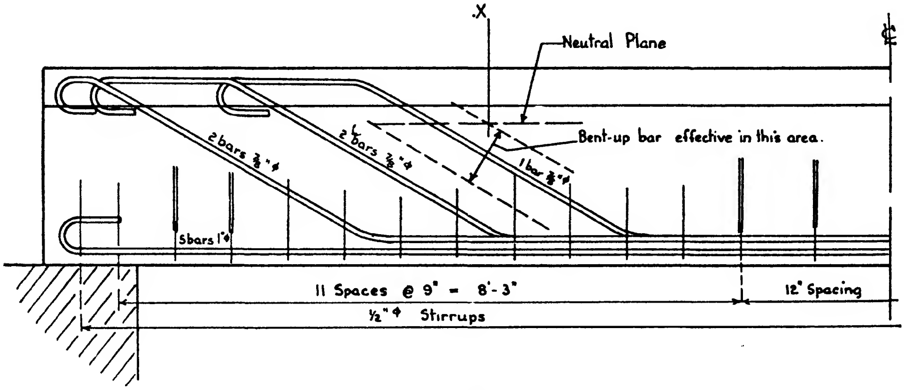



Fig. 55.

present case. At this section the bending moment (maximum) is found as follows:

Section 5 ft. from mid-span.

$$\text{D.L.M.} = 950 \times 6\tfrac{1}{4}\left(11\tfrac{1}{4} - \frac{6\frac{1}{4}}{2}\right) = 48{,}250$$

$$\text{L.L.M.} = \frac{30{,}000 \times 16\frac{1}{4} \times 6\frac{1}{4}}{22\frac{1}{2}} = 135{,}350$$

$$\text{Total B.M.} = 183{,}600 \text{ ft. lb.}$$

$$\text{The B.M. is } \frac{183{,}600}{229{,}000} = 0{\cdot}8 \times \text{the moment at mid-span.}$$

$$\text{The available steel area is } A_s = \frac{5{\cdot}13}{6{\cdot}93} = 0{\cdot}74 \times \text{that at mid-span.}$$

Since, however, the centre of gravity of the remaining bars will be lowered, affording a greater effective depth and lever arm, and the bars are bent up with a large radius a little nearer the support than this section the steel provided is satisfactory.

We must now turn our attention to the web reinforcement. The shear should be calculated at intervals of say 2 ft. along the span, and since the dead load shear at mid-span is zero, and at other points is equal to the dead load measured from mid-span to the section in question, the mid-span section is a convenient one to work from. It is necessary only to consider the shear as far as the near edge of the support, as from this point the bearing reaction will commence to reduce the shear.

The summary of shears given in the table below is prepared by considering *Fig.* 56. The dead load shear is at each section equal to $950x$, and the live load shear (see p. 29) is $30{,}000 \times \dfrac{(10 + x)}{20} = 15{,}000 + 1{,}500x$. It will be noticed that the effective span has been taken as 20 ft. for the calculation of the live load shear; it might at first sight appear more consistent to use $22\frac{1}{2}$ ft. as for bending moment, but the difference is small and the lower dimension is, for shear, on the side of safety. When a factor in design can vary between two limits

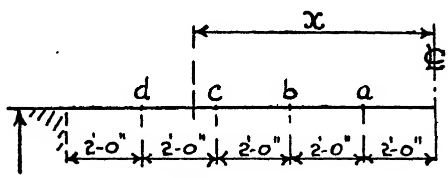



**Fig. 56.**

it is often advisable to select the more exacting value (as has been done here) each time a stress is calculated, and there is no real inconsistency in this practice.

TABLE OF SHEARS.

| Section at | D.L.V. | L.L.V. | Total V. |
|---|---|---|---|
| Mid-span . . . . . . . . . . | 0 | 15,000 | 15,000 |
| *a* . . . . . . . . . . . . . | 1,900 | 18,000 | 19,900 |
| *b* . . . . . . . . . . . . . | 3,800 | 21,000 | 24,800 |
| *c* . . . . . . . . . . . . . | 5,700 | 24,000 | 29,700 |
| *d* . . . . . . . . . . . . . | 7,600 | 27,000 | 34,600 |
| Support edge . . . . . . . . | 9,500 | 30,000 | 39,500 |

We proceed by the method outlined on p. 49, and the next step is to calculate the value in shear resistance of the pair of $\frac{7}{8}$-in. diameter bars, using equation (33). The effective depth $d$ is here taken to the bottom layer of bars.

$$V_1 = \frac{1{\cdot}43 \times 1{\cdot}2 \times 16{,}000 \times 0{\cdot}9 \times 29\frac{1}{4}}{21} = 34{,}400 \text{ lb.}$$

It is thus seen that in the region of the pairs of bent-up bars only a nominal number of stirrups need be supplied. The single bar will offer effective shear resistance of one-half this amount, or 17,200 lb. This bar crosses the neutral plane approximately at section $c$ where the shear is 29,700 lb., and the balance to be carried by stirrups, assuming all the shear to be carried by web reinforcement, is 12,500 lb., for which the stirrup spacing with $\frac{1}{2}$-in. diameter links (two legs) is

$$s = \frac{0{\cdot}39 \times 16{,}000 \times 0{\cdot}9 \times 28{\cdot}4}{12{,}500} = 12{\cdot}8 \text{ in.}$$

(If one-third of the shear is assumed to be resisted by concrete, as is legitimate when the maximum unit shear on the section is less than, say, 120 lb., the shear to be resisted by web reinforcement is 19,800 lb. and that remaining to the stirrups 2,600 lb.)

The bent-up bar may be considered effective over a portion of the beam as indicated in *Fig.* 55 ; the horizontal length of the area shown in elevation is, say, $\frac{2}{3}d$ measured half on either side of the bar. The limiting line crosses the neutral plane about $5\frac{1}{4}$ ft. from mid-span (section $X$,) at which section the total shear is 27,900 lb.

$$v = \frac{27{,}900}{12 \times 0{\cdot}9 \times 28{\cdot}4} = 91 \text{ lb. per square inch.}$$

Assuming one-third of the shear to be resisted by the concrete, that remaining to the stirrups is 18,600 lb., and the stirrup spacing is

$$s = \frac{0{\cdot}39 \times 16{,}000 \times 0{\cdot}9 \times 28{\cdot}4}{18{,}600} = 8{\cdot}5 \text{ in.}$$

By direct proportion of shears the spacing at section $b$ is

$$s = \frac{8{\cdot}5 \times 27{,}900}{24{,}800} = 9{\cdot}6 \text{ in.,}$$

and at section $a$ is $s = \dfrac{8{\cdot}5 \times 27{,}900}{19.900} = 12$ in.

In detailing we should use this 12-in. spacing for the middle 4 ft. of the beam, and then 8 or 9-in. spacing as far as the bent-up bars (section $X$ already referred to) ; from this section to the support a nominal spacing of 12 in. may be used in the absence of any more exacting requirement in the calculations. (Some designers prefer to use the spacing calculated at section $X$, throughout the region of the bent-up bars, which would mean a 9-in. spacing to the support ; the slight amount of additional steel is in many cases worth using. This method of spacing has been adopted in *Fig.* 55.)

# CHAPTER VIII

## DESIGN OF CONTINUOUS BEAMS

APART from ascertaining the bending moments and shears in a continuous beam system there is little difference in the design of the actual section from that of the simple beam. The reinforcement requirements will have been understood from the study of Chapter VI, but there are several variations in the detailing. The moments and shears are obtained by one of the methods outlined in Chapter II, and a little more will be said about this later.

Comparing the conditions governing continuous beams with those in the case of slabs we find several differences which affect the disposition of the reinforcement. In a slab the dead load is generally appreciably lighter than the live load and this means that the point of contraflexure is subject to considerable variation and in some cases reverse moment (that is, negative moment where positive moment is expected) may obtain at mid-span. This means that it is often more economical to provide straight bars in the top and bottom of the slab rather than to crank them from one face to the other ; this is the more easily done because shear in slabs is seldom of serious consequence and it is not usual to provide shear reinforcement. With beams the proportion of dead to live load increases and consequently the point of contraflexure moves about through a smaller range, and, if bent, the same bars may be used in both faces of the beam. In the last chapter the discussion on T-beams showed the reason why shear is likely to be severe in beams as compared with slabs, and bending up of the tensile reinforcement towards the supports permits this steel to be used as web reinforcement before it continues in the top face as tensile reinforcement for negative moment and is then carried on into the adjacent span for the same purpose.

There are certain difficulties. The deeper the beam the longer is the distance required for the steel to be transferred from one face to the other, but it should be remembered that the bar is serving in a very useful capacity as " shear " or diagonal tension reinforcement. Negative moment is likely to be high and a large area of steel may be required, and a little ingenuity is required to dispose the bars so that they are effective without making it too difficult for the concrete to be placed efficiently around them. The student should study *Fig.* 64 and any other drawings of reinforcement in continuous beams which may come his way and then sketch several arrangements for himself.

Since most continuous reinforced concrete beams are T-beams there is another difficulty to surmount. With negative moment at the support the flange is in tension and the lower portion of the stem or rib is in compression, and the com-

pression area is in the ordinary way insufficient. This may be dealt with by one of the following methods:

(*a*) Providing a haunch as shown in *Fig.* 57,

(*b*) Flaring out the stem as shown in *Fig.* 58.

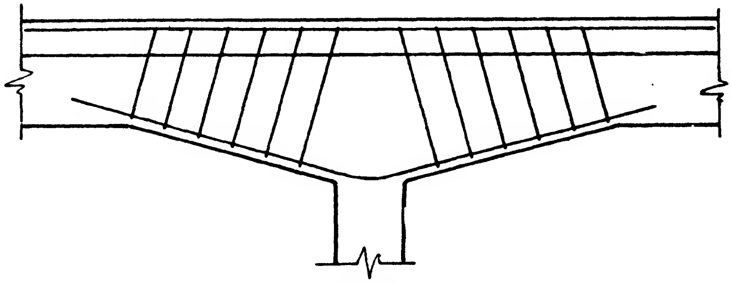


**Fig. 57.**

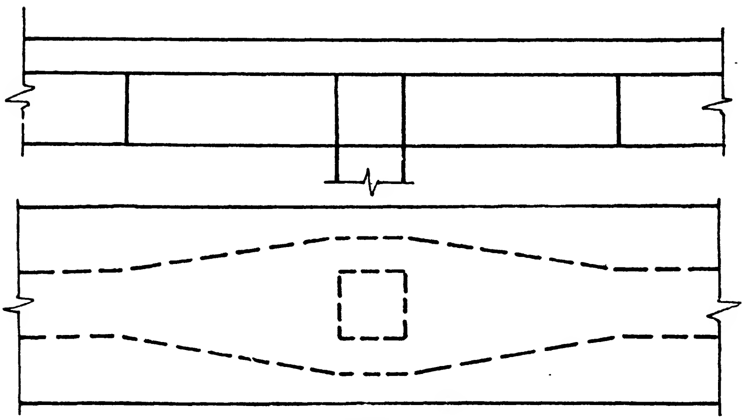


**Fig. 58.**

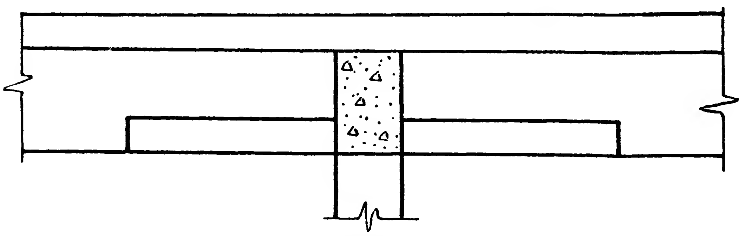


**Fig. 59.**

(*c*) Providing a lower flange as shown in *Fig.* 59, and

(*d*) Providing compression reinforcement.

The commonest methods are (*a*) and (*d*). It can be seen from the shapes of bending moment diagrams for continuous beams (*Fig.* 17, for example) that the negative bending moment falls away rapidly, and in these circumstances it is often regarded as legitimate to use somewhat higher compressive working stresses

for concrete near the support. This method has not been adopted in this book, but where it is employed some regard should be paid to the ratio of dead to live load; where the dead load is relatively high the use of a high stress might lead to appreciable plastic yield (see p. 35) possibly resulting in higher positive moments at mid-span and slight overloading of the tensile steel provided for positive moment.

With method (*a*), which can be used also in conjunction with method (*d*), the compressive stress in the concrete is assumed to be parallel to the face of the haunch and the component in a direction parallel to the tensile force in the top of the beam is reduced in the ratio of the cosine of the angle of slope. This is allowed for automatically by measuring the effective depth $d$ from the centre of gravity of the steel in the top face over the edge of the support perpendicularly to the sloping face of the haunch instead of vertically. The stirrups provided for shear reinforcement should also be inclined in this direction throughout the length of the haunch as shown in *Fig.* 57. Methods (*b*) and (*c*) need no elaboration as they can be employed and the stresses found by the ordinary rectangular and T-beam formulæ. These two methods are not popular as they complicate the formwork, and they are not recommended for ordinary cases.

Method (*d*) is useful and is frequently employed. It will first be explained and then later demonstrated by an example.

## Doubly-Reinforced Beams.

In Chapter IV (on p. 40) we saw that for fixed allowable working stresses in concrete and steel, and a given modular ratio $n$, $p$, $k$, $j$, and $R$ are determined, and therefore the moment of resistance of the section, $M$, is fixed (see equation (28)). Values for common working stresses are given in *Table* IV. The beam

TABLE IV.
For $n = 15$

| $f_s$ | $f_c$ | $R$ | $p$ | $j$ | $k$ |
|---|---|---|---|---|---|
| 14,000 | 600 | 102 | 0·0084 | 0·870 | 0·392 |
| | 650 | 115 | 0·0095 | 0·862 | 0·411 |
| | 700 | 128 | 0·0107 | 0·857 | 0·429 |
| | 750 | 142 | 0·0120 | 0·851 | 0·446 |
| | 800 | 156 | 0·0132 | 0·846 | 0·462 |
| | 850 | 170 | 0·0145 | 0·841 | 0·477 |
| | 900 | 185 | 0·0158 | 0·836 | 0·491 |
| 16,000 | 600 | 95 | 0·0068 | 0·880 | 0·360 |
| | 650 | 107½ | 0·0077 | 0·874 | 0·379 |
| | 700 | 120½ | 0·0087 | 0·868 | 0·397 |
| | 750 | 133½ | 0·0097 | 0·862 | 0·413 |
| | 800 | 147 | 0·0107 | 0·857 | 0·429 |
| | 850 | 160½ | 0·0118 | 0·852 | 0·444 |
| | 900 | 174½ | 0·0129 | 0·847 | 0·458 |
| 18,000 | 600 | 89 | 0·0056 | 0·889 | 0·333 |
| | 650 | 101 | 0·0063 | 0·883 | 0·351 |
| | 700 | 113 | 0·0072 | 0·877 | 0·368 |
| | 750 | 126 | 0·0080 | 0·872 | 0·385 |
| | 800 | 139 | 0·0089 | 0·867 | 0·400 |
| | 850 | 152 | 0·0098 | 0·862 | 0·415 |
| | 900 | 165 | 0·0107 | 0·857 | 0·429 |

section, being controlled by other factors, is determined, and the values of $p$ and $R$ suitable to the section simply reinforced for the full working stresses are found and the corresponding values of $A_s$ and $M$ calculated. At the support of a continuous T-beam the value of $b$ is that for the stem. The applied bending moment we will call $M_1$, so that the balance of moment to be resisted by additional reinforcement, both tensile and compressive, is $M_1 - M$. The lever arm between the centres of tension and compression is $(d - d_1)$, where $d_1$ is the embedment depth to the centre of the steel in the compression face * (see *Fig.* 60). If the

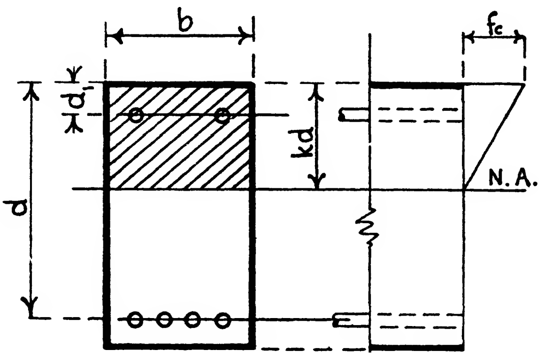



**Fig. 60.**

stress in the tensile steel is $f_s$ (usually 18,000 lb. per square inch) the additional area of tensile steel required is

$$A_t = \frac{M_1 - M}{f_s(d - d_1)} \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The stress ($f_s'$) in the compressive steel is in proportion to $f_s$ directly as their distances from the neutral axis, or

$$\frac{f_s'}{f_s} = \frac{(kd - d_1)}{(d - kd)} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

Since the additional compression, represented by $A_c \times f_s'$ must equal the additional tension, represented by $A_t \times f_s$,

$$A_c = A_t \times \frac{f_s}{f_s'} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

and the required amount of compressive reinforcement may be found directly by the equation

$$A_c = A_t \frac{(d - kd)}{(kd - d_1)} \quad . \quad . \quad . \quad . \quad . \quad (41)$$

The total sectional area of tensile reinforcement required is $A_s + A_t$, and the compressive reinforcement is $A_c$.

* The section is taken at mid-span of a rectangular beam so that compression exists above the neutral axis.

It will appear from equation (39), or from the fact that the calculated value of $f_s'$ is only $n$ times the concrete stress at the same point (closer to the neutral axis than the extreme fibre) that the compressive reinforcement is operating at a low and inefficient stress. Actually the stress is likely to be appreciably higher than that calculated. This is owing to the effect of shrinkage which throws an initial compression on the steel, and to plastic yield which builds up the compression in the steel in the manner to be explained in the next chapter. It is therefore necessary that the calculated compressive stress, which is underestimated, should be low.

Compressive reinforcement should, in order to prevent buckling, be well anchored by links spaced apart not more than $\frac{d}{2}$ or 16 bar (compression steel) diameters, whichever dimension is less, and a liberal cover of concrete should be provided.

An example demonstrating the calculation of double reinforcement is given on p. 89. With this method the compressive reinforcement should not generally exceed 1 per cent. of the sectional area of the beam.

### Doubly-Reinforced Beams with Equal Steel Top and Bottom.

Where much more than 1 per cent. of compressive reinforcement is required another method of design is adopted, the only justification for which is the fact that it has been freely employed in practice satisfactorily. The effect of the concrete on the moment of resistance is entirely neglected and the steel is designed as if it formed the flanges of a rolled steel joist. If the applied moment is $M$ then the cross-sectional area of steel required in either face is

$$A_s = \frac{M}{f_s(d - d_1)} \quad . \quad . \quad . \quad . \quad . \quad (42)$$

The value of $f_s$ is usually taken as 18,000 lb. per square inch. The steel should be well anchored by links spaced not farther apart than eight bar diameters (compression bars) or 6 in., whichever is less.

Diagonal tension and bond stresses must be adequately provided for in all beams.

### Example of Continuous-Beam Design.

A concrete floor 5 in. thick is supported by beams spaced at 12-ft. centres, the beams and slab being monolithic. The beams are carried at 20-ft. intervals by columns over a large number of bays. The wearing surface of the floor weighs 10 lb. per square foot, and the live load is 100 lb. per square foot, with an additional single concentrated rolling load of 10,000 lb. on the beam. Design the interior spans of the beam. The allowable stresses are $f_c = 750$ lb. per square inch and $f_s = 16{,}000$ lb. per square inch.

Assume that the load on the beam is taken as from centre to centre of the slab spans.

Dead load.

| | | |
|---|---|---|
| Wearing surface | $10 \times 12$ | $= 120$ |
| Slab | $5 \times 12 \times 12$ | $= 720$ |
| Beam (assumed) | $1 \times 1\frac{1}{2} \times 150$ | $= 225$ |
| | | 1,065 lb. per foot run of beam. |

Live load.

| | | |
|---|---|---|
| Distributed | $12 \times 100$ | $= 1{,}200$ lb. per foot run of beam. |
| Concentrated | | $= 10{,}000$ lb. |

Shear at support.

$$V = 10\ (1{,}065 + 1{,}200) + 10{,}000 = 32{,}650 \text{ lb.}$$

If the allowable shear is 160 lb. per square inch, the approximate effective depth is $d = 19$ in., and

$$v = \frac{32{,}650}{12 \times 0{\cdot}9 \times 19} = 159 \text{ lb. per square inch.}$$

Bending Moment at Mid-span (use *Fig.* 31).—Ignoring the effect of loads in alternate bays, the average ordinate of the influence line for the mid-span section is approximately 0·075 ; if allowance is made for live load on alternate bays the coefficient of $wl^2$ becomes 0·084. With a point load placed at mid-span the influence line ordinate is 0·171. The dead load moment is one-third of the "free" moment (see p. 58).

$$\text{D.L.M.} \qquad \frac{1}{24} \times 1{,}065 \times 20^2 = 17{,}750$$

$$\text{L.L.M.} \begin{cases} 0{\cdot}084 \times 1{,}200 \times 20^2 = 40{,}300 \\ 0{\cdot}171 \times 10{,}000 \times 20 = 34{,}200 \end{cases}$$

$$\text{Maximum mid-span moment} = 92{,}250 \text{ ft. lb.}$$

Bending Moment at Support (use *Fig.* 31).—The average ordinate of the influence line in one span is slightly in excess of 0·05, and for two adjacent spans loaded the coefficient of $wl^2$ may therefore be taken as approximately 0·10. If alternate spans thereafter are loaded this becomes about 0·115. A single concentration placed at a section $0{\cdot}4l$ from the support gives an ordinate of 0·085. The dead load moment is two-thirds of the "free" moment (see p. 58).

$$\text{D.L.M. (twice the mid-span moment)} = 35{,}500$$

$$\text{L.L.M.} \begin{cases} 0{\cdot}115 \times 1{,}200 \times 20^2 = 55{,}250 \\ 0{\cdot}085 \times 10{,}000 \times 20 = 17{,}000 \end{cases}$$

$$\text{Maximum support moment} = 107{,}750 \text{ ft. lb.}$$

These bending moments are shown in *Fig.* 61, and in drawing them it should be remembered that the shape is the same as that of the "free" moment diagram for the same loading—it is only the zero or base line which changes. For positive moment the loading is the same as already given for the mid-span moment : for negative moment the span without the concentration is selected, as the concentration in the next span causes the bending moment to fall off more rapidly.

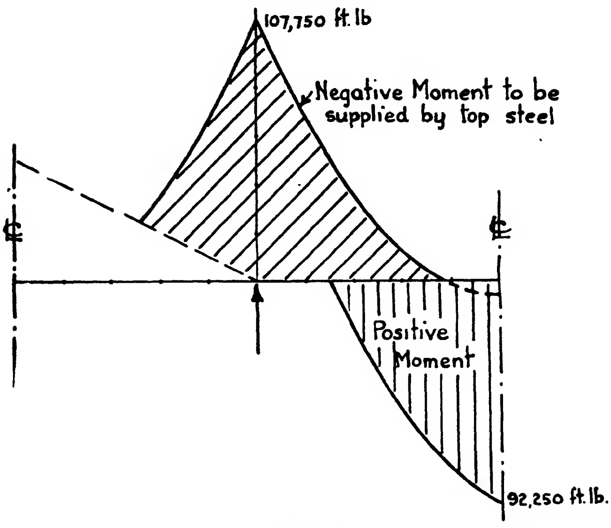



Fig. 61.

DESIGN OF MID-SPAN SECTION.—The controlling factors in obtaining the flange width in the design of the T-beam in this case are the slab thickness and the beam span (see p. 71) which both give 60 in. The assumed beam section is

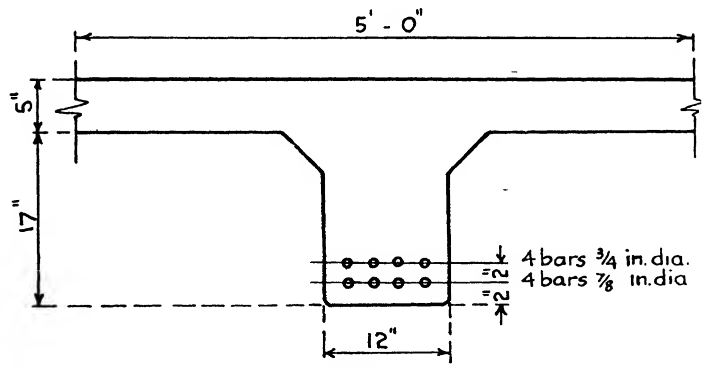



Fig. 62.

shown in *Fig.* 62, and the lever arm may be taken as approximately 17 inches (see footnote, p. 76). The approximate steel area required is

$$A_s = \frac{92{,}250 \times 12}{16{,}000 \times 17} = 4{\cdot}07 \text{ sq. in.}$$

Four $1\frac{1}{8}$-in. bars would supply 3·98 sq. in. and these might be used, but for demonstration purposes illustrating how one layer of bars may be bent up to supply negative moment, two layers will be provided and the upper layer will be used as shear reinforcement.

| | Area. | Moment about bottom face. |
|---|---|---|
| Four $\frac{7}{8}$-in. bars . . . . . . . . . | 2·40 | ($\times$ 2 in.) = 4·80 |
| Four $\frac{3}{4}$-in. bars . . . . . . . . . | 1·77 | ($\times$ 4 in.) = 7·08 |
| | 4·17 sq. in. | 11·88 |

The distance of the centre of gravity from the bottom face is $\frac{11 \cdot 88}{4 \cdot 17} = 2 \cdot 85$ in. and the lower layer of bars is 0·85 in. below the centre of gravity.

The effective depth is $d = 22 - 2 \cdot 85 = 19 \cdot 15$ in.

$$\frac{t}{d} = \frac{5}{19 \cdot 15} = 0 \cdot 261 \; ; \; p = \frac{4 \cdot 17}{60 \times 19 \cdot 15} = 0 \cdot 00363.$$

Using design chart, *Fig.* 53A,

$$k = 0 \cdot 281 \text{ and } j = 0 \cdot 907.$$

The average steel stress is

$$f_s = \frac{92{,}250 \times 12}{4 \cdot 17 \times 0 \cdot 907 \times 19 \cdot 15} = 15{,}300 \text{ lb. per square inch.}$$

The maximum steel stress occurs in the lower layer and is in proportion to the distance from the neutral axis, and as $d(1 - k) = 19 \cdot 15 \times 0 \cdot 719 = 13 \cdot 76$,

$\therefore$ Maximum steel stress

$$f_s = \frac{15{,}300 \times 14 \cdot 61}{13 \cdot 76} = 16{,}250 \text{ lb. per square inch.}$$

($1\frac{1}{2}$ per cent. of excess stress is not unreasonable.)

The maximum concrete stress is obtained from equation (23). Since we make use of $k$, which is a coefficient of $d$, the value of $f_s$ used in the equation must be that at the centre of gravity, a distance equal to $d$ below the top face.

$$f_c = \frac{15{,}300 \times 0 \cdot 281}{15 \times 0 \cdot 719} = 399 \text{ lb. per square inch.}$$

Later the steel stress would be investigated at the sections where bars are bent up from the main group.

Design of Section at the Support.—It is obvious that the allowable concrete stress would in the ordinary way be exceeded. Alternative methods of dealing with the problem will be given.

(1) Use of a haunch.—In calculating the bending moments no account has been taken of the variable moment of inertia resulting from variation in the section: the effect is small in a case such as that in the example, but it should be borne in mind that the stresses are likely to be a little higher than those calculated (see p. 19).

The required effective depth, measured at right angles to the sloping face of the haunch, can be found from formula (29)

$$d^2 = \frac{107{,}750}{133\frac{1}{2} \times 1} = (28 \cdot 4)^2.$$

This effective depth would be supplied as shown in *Fig.* 63. The real effective depth is the distance vertically from the steel, but the reduction is made when calculating concrete stress to allow for the component of compressive stress in a direction parallel to the steel (see p. 83). The slope of the haunch should be

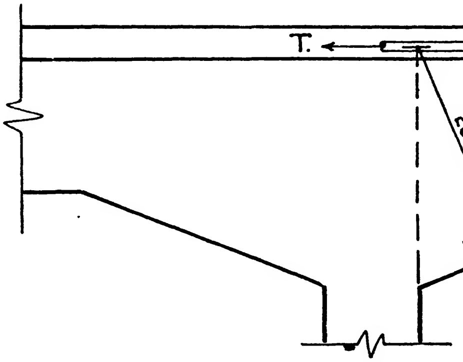



**Fig. 63.**

gradual. The effective depth for the steel is taken as the vertical depth already referred to.

Then $$A_s = \frac{107{,}750 \times 12}{16{,}000 \times 0{\cdot}9 \times 30} = 3{\cdot}0 \text{ sq. in.}$$

When no haunch is used the alternative method is:

(2) Doubly-reinforced beam (using the method described on p. 84).

$$b = 1 \text{ ft.} \qquad d = 19 \text{ in.}$$

$$M_1 = 107{,}750 \text{ ft. lb.}$$

For $R = 133\frac{1}{2}$ , $k = 0{\cdot}414$ and $kd = 7{\cdot}87$ ;

$p = 0{\cdot}0097$ ; $j = 0{\cdot}86$ ;

$A_s = 0{\cdot}0097 \times 12 \times 19 = 2{\cdot}21$ sq. in.

Corresponding $M = 133\frac{1}{2} \times 1 \times 19^2 = 48{,}300$ ft. lb.

Then $M_1 - M = 107{,}750 - 48{,}300 = 59{,}450$ ft. lb.

$d - d_1 = 19 - 2{\cdot}85$ (assumed, see p. 86) $= 16{\cdot}15$ in.

Additional tensile steel required. $A_t = \dfrac{59{,}450 \times 12}{16{,}000 \times 16{\cdot}15} = 2{\cdot}76$ sq. in.

$\therefore$ Total tensile steel $= 2{\cdot}21 + 2{\cdot}76 = 4{\cdot}97$ sq. in.

The stress in the compressive steel may be found from equation (39), but unless this is required the value of $A_c$ (sectional area of compressive steel) can be found directly from equation (41).

$$A_c = \frac{2{\cdot}76 \times (19 - 7{\cdot}87)}{(7{\cdot}87 - 2{\cdot}85)}$$

$$= \frac{2{\cdot}76 \times 11{\cdot}13}{5{\cdot}02} = 6{\cdot}12 \text{ sq. in.}$$

This is a proportionately large amount of compressive reinforcement.

Assuming the steel to be supplied in accordance with the principle of equal steel top and bottom, equation (42) would be used, from which

$$A_s \doteq \frac{107{,}750 \times 12}{16{,}000 \times 16{\cdot}15} = 5 \text{ sq. in.}$$

This sectional area of steel will be supplied in both faces. *Fig.* 64 shows the beam with the reinforcement in elevation. Two layers of bars have been provided in both top and bottom at the support section. Actually it is permissible to spread out the steel in one layer by going outside the beam width into the flange area.

The bent-up bars should be detailed from a consideration of *Fig.* 61. Steel must be provided in the bottom of the beam to resist the positive moment. This moment, as shown in *Fig.* 61, has been calculated with the concentration at mid-span only; the maximum positive moments at intermediate sections will be somewhat greater than indicated if the concentration is placed at the section

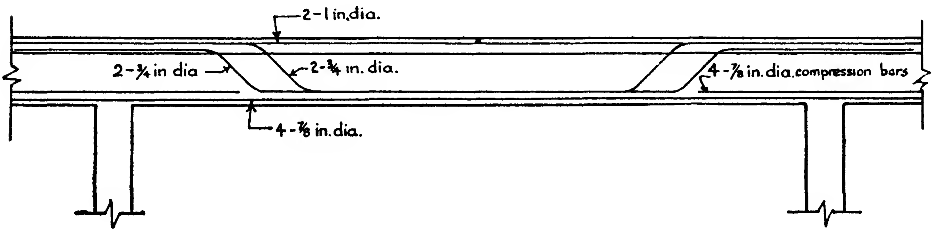



**Fig. 64.**

in question, but the additional moment at sections near mid-span will be slight, and at sections nearer the support the steel provided is generally more than ample. The maximum moment should be calculated for the sections where bars are bent up if there is any doubt about the remaining bars being adequate.

The requisite area of steel in the top at the support can be obtained by using the eight ¾-in. bars (four from each span) providing 3·54 sq. in. together with a pair of 1-in. bars (which may have to be accommodated in the flange just outside the beam width) supplying 1·57 sq. in. "Wiring" bars, on which to hang the stirrups, would be provided in addition in the middle of the beam from the points where the ¾-in. bars incline away from the top face. The arrangement shown in *Fig.* 64 supplies the necessary reinforcement for negative moment.

It is obvious from a comparison of *Figs.* 61 and 64 that there is provision for resisting the positive moment also. If any doubt is entertained the positive bending moment should be calculated at the section where the ¾-in. bars commence to bend up. The concentrated load would be placed at this section, and the bending moment calculated with the aid of *Fig.* 31.

SHEAR.—For a distance of say 3 ft. from the support the whole of the shear should be provided for by stirrups. The maximum shear is 32,650 lb. and the spacing of ½-in. stirrups with two legs is

$$s = \frac{0{\cdot}39 \times 16{,}000 \times 0{\cdot}86 \times 19}{32{,}650} = 3{\cdot}12 \text{ in.}$$

The spacing of the stirrups in the region of the bent-up bars and beyond is determined in the manner demonstrated on p. 79. The exact shears in the continuous beam can be obtained by loading the beam as explained on p. 29, calculating the moments and thence the shears as explained on p. 20. It is, however, scarcely necessary to do this, as the maximum shears will be only slightly different from those in the simple beam; the shears may therefore be calculated in this way and the stirrups provided rather liberally. At the first interior support, a span's length from the free end, the shear will be possibly from 20 to 25 per cent. greater than that calculated for a freely-supported beam.

## CHAPTER IX

## DESIGN OF COLUMNS

CONCRETE is particularly suitable for columns. Here its compressive resistance can be developed to the full and the whole section can act at maximum efficiency. In a column there are two factors to be considered: (*a*) the direct crushing strength of the material composing the column, and (*b*) the tendency of a strut or column under load to buckle sideways. Of the two the second more often controls the design, and we will consider this factor first.

### Buckling Tendency.

If a long, thin cane be held by the top vertically and its bottom end be thrust hard against the ground the cane will bend, and if the pressure is maintained the " bow " will increase until the cane snaps under a load which would have caused no damage had the cane been held in position vertically at a number of intermediate points and thus been prevented from " buckling." The buckling is due to the fact that the thrust line can never be made to coincide exactly with the axis of the member, and thus a bending moment is introduced; any movement sideways increases the eccentricity of the applied load and hence the bending moment, and failure is progressive. So with any long, thin column; if it be unstayed laterally it will buckle sideways and fail under a smaller load than would cause it to fail were the column reduced to the proportions of a short cylinder having the same cross-sectional area. When a thin column is stayed laterally by being held in position at a number of intermediate points the effect is to convert it into a series of short cylinders and the strength is thereby restored. The condition of support of the column at either end also enters into consideration. If the two ends are free to incline at the least tendency we have what are termed pin joints, and the end condition offers no resistance to the buckling tendency. If, however, the ends are held rigidly so that any tendency to inclination is resisted the column is said to have fixed ends, and this condition offers a certain amount of resistance to buckling. The susceptibility of a column to buckling is inversely proportional to its stiffness.

Whatever value is decided upon for the strength of the material composing the column, there is a limiting length of column to which it is considered applicable, and above this length the allowable working stress is reduced. It is beyond the limits of this book to go into the question of the reduced allowable stresses in long columns. In concrete work the column is usually made of such proportions as to be suitable for taking, if necessary, the maximum stress allowable

for the concrete as a material without any reduction for slenderness. A suitable maximum length is fifteen times the least thickness; or put in another way (since the length is controlled) the least width of cross section should be $\frac{1}{15} \times$ the length, and preferably it should be not less than one-twelfth. This applies to columns with the average amount of end fixity obtained by framing in beams and slabs in ordinary building construction. This length is referred to as the unsupported length, and where bracing is introduced as sometimes in the legs of water towers, the unsupported length is taken for one tier only. In this case the bracing must be introduced so as to stiffen the column in all directions and not merely to hold it as a pin joint, or stiff in one plane only. If a brace holds a column stiffly in one plane the unsupported length of the column in that plane is taken for the one tier, and the thickness of section in the same plane is related to it; in the other plane the longer unstayed length requires a deeper section.

## Allowable Stress.

For columns of the proportions already referred to the full allowable stress for concrete in direct compression is permitted to be used. The crushing strength of concrete was discussed on p. 38 and the statement was made that the ultimate strength of a column was about 0·7 × the crushing strength of the test cube, plus the strength of the reinforcing steel. This was compared with the compressive strength of concrete in a beam, which was from 1·0 to 1·2 × the cube strength. For this reason a 20 per cent. lower average working stress is proposed in members subjected to direct compression. On the scale of stresses proposed on p. 38 the corresponding allowable average concrete stresses in columns are those given in the following table.

Table V.

Average Stress in Reinforced Concrete Columns, corresponding to, and in conjunction with, Stresses given in Table I.

| Concrete mix. | | | Average unit working stress in columns $f_c$ | Modular ratio. | Crushing strength of concrete in 6-in. cubes. | |
|---|---|---|---|---|---|---|
| | | | | | At 28 days with ordinary Portland cement. At 7 days with rapid-hardening Portland cement. | An additional test (if required) as an indication, at 7 days with ordinary Portland cement, at 3 days with rapid-hardening Portland cement, should give results as below. |
| Cement. lb. | Fine aggregate. cb. ft. | Coarse aggregate. cb. ft. | lb. per sq. in. | $n$ | lb. per sq. in. | lb. per sq. in. |
| A | : 2 | : 4 | 4A + 240 | — | 15A + 900 | 10A + 600 |
| 90 | : 2 | : 4 | 600 | 15 | 2,250 | 1,500 |
| 120 | : 2 | : 4 | 720 | 15 | 2,700 | 1,800 |
| 150 | : 2 | : 4 | 840 | 12 | 3,150 | 2,100 |
| 180 | : 2 | : 4 | 960 | 10 | 3,600 | 2,400 |

Stresses under combined bending and compression should also conform to the stresses given in Table I.

The stress is calculated on the equivalent sectional area of the column which, referred to *Fig.* 65, is

$$ab + (n - 1) A_s \quad . \quad . \quad . \quad . \quad . \quad (43)$$

where $A_s$ is the total cross-sectional area of the bars. In buildings liable to fire the concrete outside the "core" (the area bounded by the reinforcement) is considered to be protective only and the core area replaces $ab$ in formula (43).

The vertical bars are provided mainly to resist slight bending moments which may be unintentionally introduced, but their compressive strength is calculated on the assumption that the unit steel stress is $n$ times the stress in the surrounding concrete. This neglects the initial stress in the steel due to the shrinkage of the

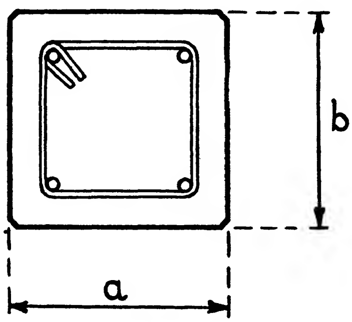



**Fig. 65.**

concrete and the stress which is subsequently built up due to "creep" or "plastic yield" (see p. 35); the actual stress in the steel is likely to be appreciably higher than the calculated stress. This, however, is not likely to be dangerous as at a certain stage (at a stress somewhat below 30,000 lb. per square inch) the steel yields and any further increase of load is thrown on the concrete. Under very high stress the increased diameter of the bar causes a bursting pressure on the surrounding concrete. At all stages there is a tendency for the vertical bars to buckle in the manner described on p. 92, and this must be resisted by a liberal supply of links or ties. These should be detailed so as to provide a stress component anchoring the bar inwards from the concrete face; any of the details shown in *Fig.* 66 is satisfactory. The links may suitably be $\frac{1}{4}$-in. bars, and the maximum spacing should be about one-half the least thickness of the column.

Columns are most frequently made square in section but may be rectangular, octagonal, or round. The total sectional area of vertical bars should be between 1 per cent. and 5 per cent. of the sectional area of the concrete. Where the columns extend from floor to floor the splicing of the bars should be made immediately above floor level.

If it becomes necessary to use very high stresses in a column additional reinforcement is provided by hooping or spirals which resists the bursting tendency of the column and adds considerably to its strength. The proportions of hooping and the calculated increase of strength are generally controlled by local regulations which vary from place to place. Most regulations governing the calculation of these increased stresses require a volume of spiral steel at least equal to 1 per cent. of

the concrete volume, and limit the pitch so that it shall not exceed one-sixth of the diameter of the hooped core, or 3 in., whichever is less.

The most reliable factor however in obtaining a strong column is a good, rich, and dense concrete, and this is more dependable than the employment of large vertical bars.

An example of column design for concentric loading will now be given.

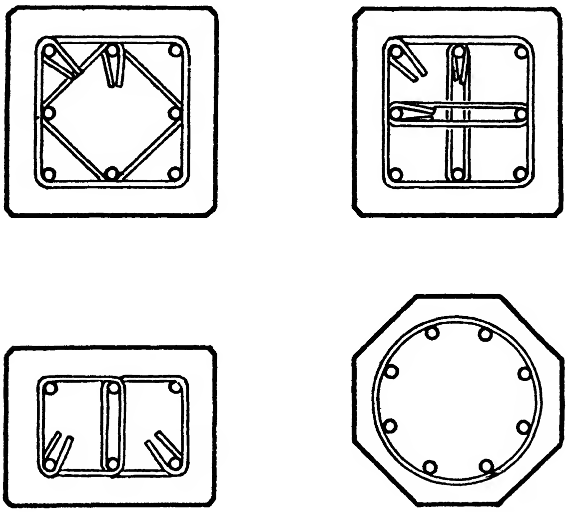


Fig. 66.

Example.—Design a reinforced concrete column to carry the floor described in the example given on p. 85. The vertical distance from floor level to floor level is 15 ft.

Dead load from floor (refer to p. 86)

$$1{,}065 \times 20 = 21{,}300 \text{ lb.}$$

Live load

$$1{,}200 \times 20 = 24{,}000$$
$$\text{Concentrated Load} = 10{,}000$$
$$\overline{\phantom{00}34{,}000 \text{ lb.}}$$

The total load applied at the top of the column is 55,300 lb. The unsupported length of column (distance from the beam soffit to the lower floor level) is approximately 13 ft. and the column should therefore be, if stress permits (which it obviously does), say 12 in. square; the vertical reinforcement should lie between 1 per cent. and 5 per cent. of the section or between 1·44 sq. in. and 7·2 sq. in.

The dead load of the column is $13 \times 150 = 1{,}950$ lb., and the maximum load is therefore 57,250 lb. From formula (43)

$$f_c = \frac{57{,}250}{144 + 14A}.$$

Since the column section has been controlled by considerations of stiffness rather than stress, the percentage of vertical reinforcement need not be high, and four $\frac{3}{4}$-in. bars (1·767 sq. in.) will suffice.

$$\therefore f_c = \frac{57{,}250}{144 + 14(1{\cdot}767)} = 340 \text{ lb. per square inch.}$$

This stress is low and the column might reasonably be made a little more slender, but before this is decided upon consideration should be given to the facts (*a*) that the beams framing into the columns at the top are 12 in. wide, and (*b*) that slight bending moments will be introduced by live load when applied to one span only. Eccentric loading will not be serious and can take place only when the column is not carrying the maximum load. Consideration (*a*) may be of sufficient importance to warrant retention of the 12-in. section.

Care will be needed in detailing the longitudinal bars to avoid fouling the bottom bars of the beams: with the light loading these columns have to sustain and their comparative freedom from bending moment the bars may be set well inside the section so as to pass inside the outermost bars in the bottom of the beam. In other circumstances it might be necessary to pass these bars on the outside, or to crimp the column bars, or splice them.

The links provided for these columns would be made from $\frac{1}{4}$-in. bars and would be spaced at 6-in. centres, except for about 18 in. or 2 ft. at the top and bottom, where the spacing would be 3 in.

## Eccentric Loading.

Consider the effect of bringing a vertical load on a column out of line with its axis. A bending moment is introduced equal to the product of the load and the distance from its line of application to the column axis. This distance is termed the eccentricity, and is denoted by $e$. The bending moment will result in stresses which can be calculated by formula (10) on p. 11. In the case of concrete columns this formula can be applied, using the equivalent moment of inertia $I_c + (n - 1)I_s$. The stresses so obtained are then superimposed on those resulting from the direct load as calculated in the last example. When, however, the eccentricity is appreciable the compressive stress resulting from the direct load is insufficient to counterbalance the tensile stresses produced by the bending moment and the net result on one side of the column is tension. This does not matter if the tension is low (up to one-quarter, say, of the allowable compressive stress) and plenty of steel reinforcement has been provided, but when the net tension is high other methods of calculation must be adopted. One such method will be described later.

The condition when one face is just unstressed, the neutral axis falling at the edge of the section, is reached when the eccentricity (see *Fig.* 67, where the two sets of forces, $P$ acting at a distance $e$ from the axis, and $P$ along the axis plus $M$, are equivalent) is exactly one-sixth of the full width of the section or at the edge of what is termed "the middle third." This can be understood quite easily if the student will apply what he reads on p. 112 dealing with bearing pressures below footings to the case of a column section—the underlying principle is exactly the same in both cases.

Corresponding to equation (10) on p. 11 we may use a formula which superimposes the stresses due to direct load transferred to the axial position and bending due to the eccentricity

$$p = \frac{P}{A} \pm \frac{My}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

In the case of a reinforced concrete section $p$ becomes $f_c$ and $A$ and $I$ are in terms of the concrete equivalents. An example is given below.

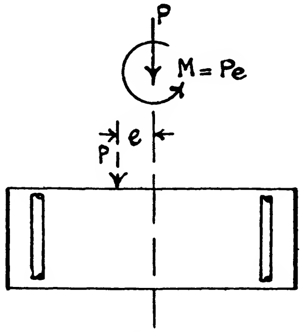



Fig. 67.

EXAMPLE.—In one bay it is necessary for the column axis in the previous example to be offset from the centre line of the beam it supports by a distance of 4 in. Calculate the maximum stresses in steel and concrete, ignoring the stiffening effect of the floor and assuming no additional reinforcement is provided. (In practice the column should be strengthened so as to be relatively as strong as the other columns.) Assume the embedment from the face of the concrete to the centre of the bar is 2 in.

Direct load applied to column top $= 55{,}300$ lb.
Bending moment $= 55{,}300 \times 4$ $= 221{,}200$ in. lb.
Equivalent area $A = 144 + 14(1{\cdot}767) = 168{\cdot}7$ sq. in.

Equivalent moment of inertia (ignore the small moments of inertia of the bars about their own axes)

Concrete $\frac{1}{12} \times 12 \times 12^3 = 1{,}728$
Steel $14(1{\cdot}767 \times 4^2) = 396$

Total $= 2{,}124$ in.$^4$

With the dead load of the column included $P$ becomes 57,250 lb.

$$\therefore f_c = \frac{57{,}250}{168{\cdot}7} \pm \frac{221{,}200 \times 6}{2{,}124} = 340 \pm 625 \text{ lb. per square inch.}$$

Maximum compressive stress in concrete = 965 lb. per square inch and maximum tensile stress in concrete = 285 lb. per square inch.

The steel stresses are proportional to the distances from the neutral axis and are obtained by multiplying the corresponding concrete stresses by the

modular ratio. Referring to *Fig.* 68, the stress increment from face to face is $\frac{285 + 965}{12} = 104$ lb. per square inch. Therefore the tensile stress in the steel is $15(285 - 208) = 1{,}155$ lb. per square inch, and the compressive stress in the steel is $15(965 - 208) = 11{,}355$ lb. per square inch.

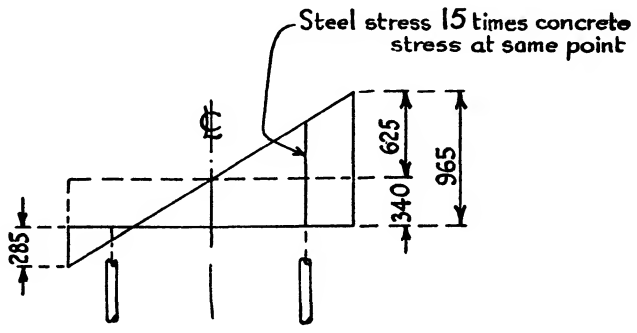



**Fig. 68.**

Owing to the reasons explained on p. 94 a further compression will, in fact, be superimposed on these calculated stresses, so that the tensile stress will be less than that calculated and the compressive stress more.

## Combined Bending and Compression.

When the eccentricity of thrust in any member is comparatively great, as is the case when the thrust line lies appreciably outside the middle-third, the imposed stresses cause the concrete to crack at the tension face, so rendering a portion of the sectional area ineffective. (If the thrust is very small the tension may be low notwithstanding the large eccentricity, but this is an exceptional case which can be dealt with by the ordinary method previously described.) The condition is somewhat similar to that of a simply-reinforced beam in which the resistance of the concrete to tension is neglected because it is incapable of sustaining the high stresses which would be imposed.

It would not be correct to determine the stresses first as a beam under bending and then to superimpose the average direct stress, the reason being that although the concrete must be considered incapable of sustaining tension, yet relief of tension by neutralisation with an equal amount of compression will obviously occur ; the design as a beam ignores the tension which would later be relieved, and analysis by this means will therefore not give correct results.

The condition of stress in the beam is a little complicated to analyse, but if we limit our consideration to sections in which the steel reinforcement is symmetrical on both faces (and this covers practically all cases generally encountered), the comparatively simple method now to be described will give a solution.

The condition is shown in *Fig.* 69 where the applied force is $P$ acting at a distance $e$ from tbe gravity axis. The internal forces reacting to $P$ are :

(1) Compression due to concrete and steel on the compression side of the neutral axis, and

(2) Tension due to the steel on the tension side of the neutral axis.

Here $p$ denotes the steel ratio calculated from the total steel and the whole cross section, so that the steel area in each face is $p\frac{bh}{2}$. It should be noted that $k$ is a coefficient of $h$, the full thickness, and not of $d$ as in the simply reinforced beam : this notation is used for simplification.

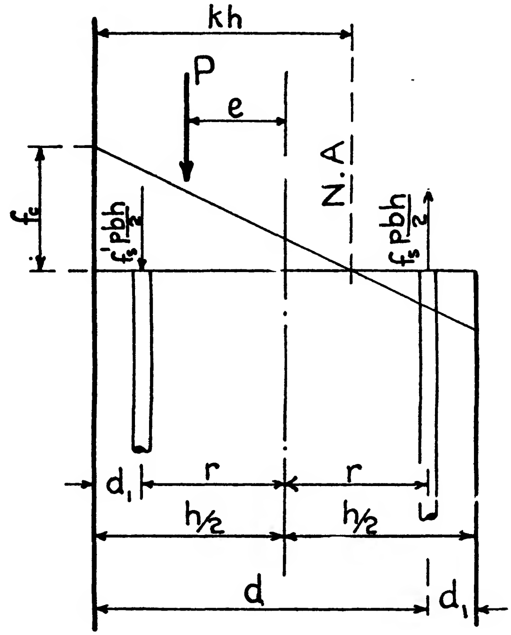



**Fig. 69.**

The total compression corresponding to (1) is :

(a) Due to concrete,* $f_c\frac{bkh}{2}$

(b) Due to steel, $f_s'\frac{pbh}{2}$.

(* The steel displaces a small amount of concrete, but this fact is quite reasonably neglected, and no deduction of area is made.)

(c) Due to steel, $f_s\frac{pbh}{2}$.

Since the algebraic sum of (a), (b), and (c) is equal to the applied force $P$, we obtain the following equation :

$$P = f_c\frac{bkh}{2} + f_s'\frac{pbh}{2} - f_s\frac{pbh}{2} \quad . \quad . \quad . \quad . \quad . \quad (45)$$

As stress is proportional to distance from the neutral axis, the stress in the concrete at the position of the compression steel is $f_c.\dfrac{kh - d_1}{kh}$, and $f_s'$ is therefore $n$ times this value, or

$$\text{(compression steel) } f_s' = nf_c\frac{kh - d_1}{kh} \quad . \quad . \quad . \quad (46)$$

By similar reasoning

$$\text{(tension steel) } f_s = nf_c\frac{d - kh}{kh} \quad . \quad . \quad . \quad . \quad (47)$$

If these values for $f_s'$ and $f_s$ are substituted in equation (45) and the resulting expression reduced, we obtain the following:

$$P = f_c\frac{bh}{2}\left[k + 2np - \frac{np}{k}\right] \quad . \quad . \quad . \quad . \quad (48)$$

from which,

$$f_c = \frac{2P}{bh\left[k + 2np - \dfrac{np}{k}\right]} \quad . \quad . \quad . \quad . \quad (49)$$

The applied bending moment is $Pe$, and may be equated to the sum of the moments of the internal forces taken about any point, which for convenience is chosen at the gravity axis.

The moments from the compression side are

(*a*) Due to concrete, $f_c\dfrac{bkh}{2}\left[\dfrac{h}{2} - \dfrac{kh}{3}\right]$

(*b*) Due to steel, $f_s'\dfrac{pbh}{2}.r.$

The moment from the tensile side is

(*c*) Due to steel, $f_s\dfrac{pbh}{2}.r.$

The sum of these three moments is equal to $Pe$, so that

$$Pe = f_c\frac{bkh}{2}\left[\frac{h}{2} - \frac{kh}{3}\right] + \frac{pbh}{2}.r(f_s' + f_s) \quad . \quad . \quad . \quad (50)$$

By substituting values previously obtained for $f_s'$ and $f_s$ and reducing the resulting expression we obtain:

$$Pe = f_cbh\left[\frac{kh}{12}(3 - 2k) + \frac{npr^2}{kh}\right] \quad . \quad . \quad . \quad . \quad (51)$$

If the right-hand side of equation (48) is multiplied by $e$, it must be equal to the right-hand side of equation (51). Both sides of the equation so obtained may be divided by $f_cbh^2$, and taking all terms over to one side of the equation we get:

$$\frac{e}{2h}\left[k + 2np - \frac{np}{k}\right] - \left[\frac{k}{12}(3 - 2k) + \frac{npr^2}{kh^2}\right] = 0 \quad . \quad . \quad (52)$$

All the terms in this expression are known except $k$. The object is, by trial and error, to determine a value of $k$ which will satisfy equation (52).

For the easy solution of this equation *Tables* VIA, VIB, and VIC are provided, corresponding to modular ratios of 15, 12, and 10.

Each line in the table gives the value for the left-hand side of equation (52) for a certain value of $k$. The simplest expression is that given for $k = 0{\cdot}5$, and this should be evaluated first. If the result is positive the expression should then be evaluated for $k = 0{\cdot}3$, or if negative, then for $k = 0{\cdot}7$. A third evaluation should then be made, the value of $k$ being selected from a consideration of the two results previously obtained. The three results allow a small curve to be plotted on squared paper which indicates the value of $k$ when the expression is equal to zero.

The true value of $k$ should then be substituted in equation (49) to give the maximum compressive stress in the concrete:

$$f_c = \frac{2P}{bh\left[k + 2np - \frac{np}{k}\right]} \qquad . \quad . \quad . \quad . \quad . \quad (49)$$

and the value of the tensile stress in the steel will be given by equation (47):

$$f_s = nf_c\frac{d - kh}{kh} \qquad . \quad . \quad . \quad . \quad . \quad (47)$$

In making these calculations for the determination of $k$ the strictest accuracy is necessary, particularly in applying it to equation (49). The denominator of this equation is the difference between $(k + 2np)$ and $\left(\frac{np}{k}\right)$; this difference is usually fairly small compared with either quantity, particularly when $k$ itself is small, so that an error in determining $k$, or in applying it in equation (49), can have a serious effect on the result.

An inspection of *Table* VIA shows that the term $\frac{pr^2}{h^2}$ is several times repeated, and its value, which is constant should therefore be tabulated to save time.

An example of the method will now be given.

EXAMPLE.—The tensile stress in the concrete obtained by working the example given on p. 97 was rather high, and it would in many cases be necessary to calculate the stresses on the assumption that the concrete had cracked at the tension face. The same example will therefore be worked by the latter method.

Data.

Concrete column of square section, 12 in. by 12 in.

Reinforcement, four $\frac{3}{4}$-in. bars.

Embedment, from the face of the concrete to the centre of the bar, 2 in.

$P = 57{,}250$ lb. $\qquad e = 4$ in.

$p = \frac{1{\cdot}767}{12 \times 12} = 0{\cdot}01227.$ $\qquad r = 4$ in.

$$\frac{pr^2}{h^2} = \frac{0{\cdot}01227 \times 4^2}{12^2} = 0{\cdot}0013632.$$

## Table VIa.

$\underline{n = 15}$ Note: k is a coefficient of h

$$p = \frac{\text{Total } A_s}{bh}$$

Equation becomes $\frac{e}{2h}\left[k + 30p - 15\frac{p}{k}\right] - \left[\frac{k}{12}(3-2k) + 15\frac{br^2}{kh^2}\right] = 0$

| k | | | | | |
|---|---|---|---|---|---|
| If k = 0·2 | $\frac{e}{2h}\left[0.2 - 45p\right]$ | − | $\left[0.0433 + 75\frac{br^2}{h^2}\right]$ | = | A |
| 0·3 | $\frac{e}{2h}\left[0.3 - 20p\right]$ | − | $\left[0.06 + 50\frac{br^2}{h^2}\right]$ | = | B |
| 0·4 | $\frac{e}{2h}\left[0.4 - 7.5p\right]$ | − | $\left[0.0733 + 37\tfrac{1}{2}\frac{br^2}{h^2}\right]$ | = | C |
| 0·5 | $\frac{e}{4h}$ | − | $\left[0.0833 + 30\frac{br^2}{h^2}\right]$ | = | $D^{\text{if }+}_{\text{if }-}$ |
| 0·6 | $\frac{e}{2h}\left[0.6 + 5p\right]$ | − | $\left[0.09 + 25\frac{br^2}{h^2}\right]$ | = | E |
| 0·7 | $\frac{e}{2h}\left[0.7 + 8.57p\right]$ | − | $\left[0.0933 + 21.43\frac{br^2}{h^2}\right]$ | = | F |
| 0·8 | $\frac{e}{2h}\left[0.8 + 11.25p\right]$ | − | $\left[0.0933 + 18.75\frac{br^2}{h^2}\right]$ | = | G |

$$f_c = \frac{2P}{bh\left[k + 2np - \frac{np}{k}\right]}$$

$$f_s = nf_c \times \frac{d - kh}{kh}$$

## Table VIb

$\underline{n = 12}$ Note: k is a coefficient of h

$$p = \frac{\text{Total } A_s}{bh}$$

Equation becomes $\frac{e}{2h}\left[k + 24p - 12\frac{p}{k}\right] - \left[\frac{k}{12}(3-2k) + 12\frac{br^2}{kh^2}\right] = 0$

| k | | | | | |
|---|---|---|---|---|---|
| If k = 0·2 | $\frac{e}{2h}\left[0.2 - 36p\right]$ | − | $\left[0.0433 + 60\frac{br^2}{h^2}\right]$ | = | A |
| 0·3 | $\frac{e}{2h}\left[0.3 - 16p\right]$ | − | $\left[0.06 + 40\frac{br^2}{h^2}\right]$ | = | B |
| 0·4 | $\frac{e}{2h}\left[0.4 - 6p\right]$ | − | $\left[0.0733 + 30\frac{br^2}{h^2}\right]$ | = | C |
| 0·5 | $\frac{e}{4h}$ | − | $\left[0.0833 + 24\frac{br^2}{h^2}\right]$ | = | D |
| 0·6 | $\frac{e}{2h}\left[0.6 + 4p\right]$ | − | $\left[0.09 + 20\frac{br^2}{h^2}\right]$ | = | E |
| 0·7 | $\frac{e}{2h}\left[0.7 + 6.86p\right]$ | − | $\left[0.0933 + 17.14\frac{br^2}{h^2}\right]$ | = | F |
| 0·8 | $\frac{e}{2h}\left[0.8 + 9p\right]$ | − | $\left[0.0933 + 15\frac{br^2}{h^2}\right]$ | = | G |

$$f_c = \frac{2P}{bh\left[k + 2np - \frac{np}{k}\right]}$$

$$f_s = nf_c \times \frac{d - kh}{kh}$$

## Table VIc.

$\underline{n = 10}$ Note: k is a coefficient of h

$$p = \frac{\text{Total } A_s}{bh}$$

Equation becomes $\frac{e}{2h}\left[k + 20p - 10\frac{p}{k}\right] - \left[\frac{k}{12}(3-2k) + 10\frac{br^2}{kh^2}\right] = 0$

| k | | | | | |
|---|---|---|---|---|---|
| If k = 0·2 | $\frac{e}{2h}\left[0.2 - 30p\right]$ | − | $\left[0.0433 + 50\frac{br^2}{h^2}\right]$ | = | A |
| 0·3 | $\frac{e}{2h}\left[0.3 - 13.33p\right]$ | − | $\left[0.06 + 33.33\frac{br^2}{h^2}\right]$ | = | B |
| 0·4 | $\frac{e}{2h}\left[0.4 - 5p\right]$ | − | $\left[0.0733 + 25\frac{br^2}{h^2}\right]$ | = | C |
| 0·5 | $\frac{e}{4h}$ | − | $\left[0.0833 + 20\frac{br^2}{h^2}\right]$ | = | D |
| 0·6 | $\frac{e}{2h}\left[0.6 + 3.33p\right]$ | − | $\left[0.09 + 16.67\frac{br^2}{h^2}\right]$ | = | E |
| 0·7 | $\frac{e}{2h}\left[0.7 + 5.71p\right]$ | − | $\left[0.0933 + 14.29\frac{br^2}{h^2}\right]$ | = | F |
| 0·8 | $\frac{e}{2h}\left[0.8 + 7.5p\right]$ | − | $\left[0.0933 + 12.5\frac{br^2}{h^2}\right]$ | = | G |

$$f_c = \frac{2P}{bh\left[k + 2np - \frac{np}{k}\right]}$$

$$f_s = nf_c \times \frac{d - kh}{kh}$$

From *Table* VIA.

If $k = 0{\cdot}5$

$$\frac{4}{4 \times 12} - \left[0{\cdot}0833 + 0{\cdot}04090\right] = -\ 0{\cdot}0409.$$

If $k = 0{\cdot}7$

$$\frac{4}{24}\left[0{\cdot}7 + 0{\cdot}105\right] - \left[0{\cdot}0933 + 0{\cdot}0292\right] = +\ 0{\cdot}0117.$$

If $k = 0{\cdot}6$

$$\frac{4}{24}\left[0{\cdot}6 + 0{\cdot}06135\right] - \left[0{\cdot}09 + 0{\cdot}03408\right] = -\ 0{\cdot}01385.$$

The curve is plotted on *Fig.* 70, from which it is seen that $k = 0{\cdot}654$.

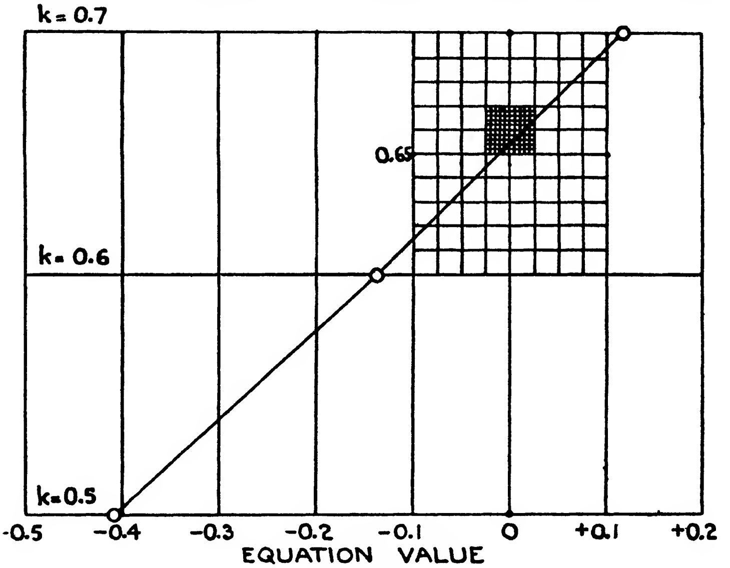



**Fig. 70.**

This value is substituted in equation (49), from which

$$f_c = \frac{2 \times 57{,}250}{12^2\left[0{\cdot}654 + 30 \times 0{\cdot}01227 - \dfrac{15 \times 0{\cdot}01227}{0{\cdot}654}\right]}$$

$$= \frac{2 \times 57{,}250}{12^2 \times 0{\cdot}740} = 1{,}073 \text{ lb. per square inch (compression).}$$

$$f_s = 15 \times 1{,}073\frac{(10 - 7{\cdot}85)}{7{\cdot}85} = 4{,}410 \text{ lb. per square inch (tension).}$$

# CHAPTER X

## DESIGN OF COLUMN FOOTINGS

THE load carried by a column has eventually to be transferred to the ground, and since the ground has nothing like the same resisting power as concrete (unless indeed it be rock) it becomes necessary to spread the load over an area appreciably larger than that of the column section. This is done by swelling out the base of the column into the form of a square or rectangular block.

If the column load is light or if the bearing capacity of the ground is high, a solid block of mass concrete having small offsets from the column face but being proportionately thick, is suitable; this type is shown in *Fig.* 71: the area to be

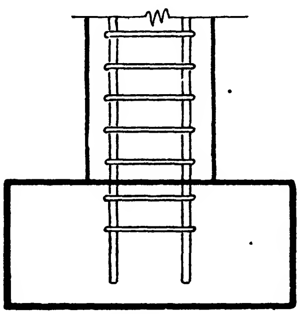


**Fig. 71.**

provided depends on the bearing capacity of the ground, and the thickness should be made such that the ratio of the offset (or projection from the column face) to the thickness is about $\frac{2}{3}$. The top face may be sloped down if desired.

When the load on the footing is high and the bearing capacity of the ground is low, the area of the footing required is such that it becomes uneconomical to provide a mass concrete footing which would in the circumstances have to be very thick. In such a case a reinforced concrete footing should be designed. This type of footing is illustrated in *Fig.* 72, and the following method is referred to that diagram.

The first step is to decide on the necessary area, and for this we must know the capacity of the ground which is discussed briefly on p. 107.

(1) The allowable unit bearing pressure $= \frac{P}{b^2}$, where $P$ is the total load to be carried, including the weight of the column and footing; from this equation we determine $b$.

(2) The column footing must not be so thin that there is a danger of the column punching through it, and one of the factors governing the thickness is therefore the punching shear. Assuming failure to occur in this way, the area over which a punching shear failure would have occurred would be $4hd$. If $p$ represents the bearing pressure per square foot (after deducting the weight of the

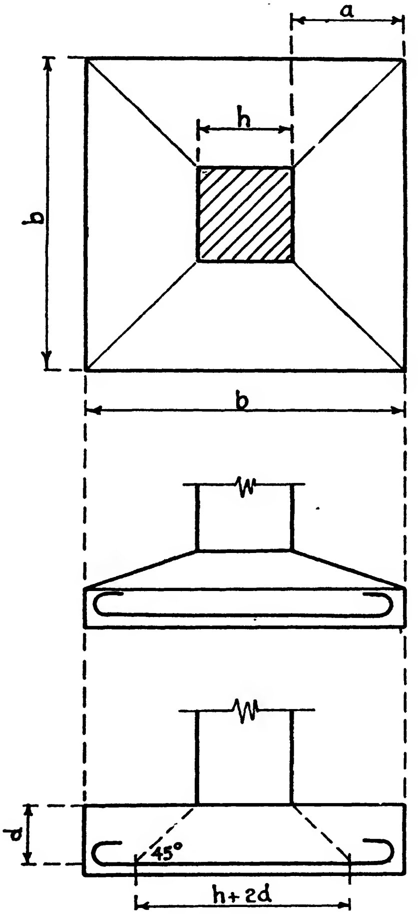



**Fig. 72.**

footing itself), then the load tending to force the footing over the column as a collar would be $p(b^2 - h^2)$. The unit punching shear is

$$s = \frac{p(b^2 - h^2)}{4hd} \qquad . \quad . \quad . \quad . \quad . \quad (53)$$

The allowable value for $s$ is usually assumed to be 0·3 of the allowable working stress of concrete in compression. It is very unusual for column footings to fail by punching shear.

(3) There is, however, another type of " shear " to consider—the diagonal

tension referred to on p. 47. The threatened plane is that lying at an angle 45 deg. from the column face and shown by the dotted line in the lowest portion of *Fig.* 72. Shear calculated on the vertical planes passing through the line of intersection of the 45-deg. planes with the bottom bars is taken as the measure of the diagonal tension, and the unit shear on the concrete should be not greater than one-tenth of the allowable compressive stress in the concrete (see p. 39). Stirrups are not provided in footings, and the concrete should be thick enough to satisfy the requirement mentioned. The shear can be worked out in the same manner as has been indicated above for punching shear, and if $V$ is the total shear per foot of width in the footing we have (in ft. units)

$$V = p\left[\frac{b^2 - (h + 2d)^2}{4(h + 2d)}\right] \quad . \quad . \quad . \quad . \quad (54)$$

The maximum unit shear then becomes (see equation (30), p. 47)

$$v = \frac{V}{12jd} \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

where $d$ is in inches, and $v$ in lb. per square inch.

(4) The bending moment must be calculated in the footing slab at the face of the column. Consider the portion of footing allotted by symmetry to one column face. Its area may be considered as a rectangle of area $ab$ less two triangles which together have an area $a^2$. The centre of gravity of the bearing area of the rectangle is at a distance $\frac{a}{2}$ from the column face, and that of the two triangles at distances $\frac{a}{3}$, so that the bending moment operating in the footing over the width $h$ at the column face is

$$M = p \times ab \times \frac{a}{2} - p \times a^2 \times \frac{a}{3} \text{ in ft. lb.-units}$$

$$= pa^2\left[\frac{b}{2} - \frac{a}{3}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

The total cross-sectional areas of the reinforcing bars provided for such a side would then be

$$A_s = \frac{M \times 12}{f_s jd} \quad . \quad . \quad . \quad . \quad . \quad (57)$$

The compression in the concrete may be assumed to be spread over a width equal to $(h + 2d)$, and the reinforcing steel calculated from equation (57) may be spread over this effective width, additional bars at a nominal spacing (say twice that selected within the main strip) being provided outside these limits. The required steel per foot of width and the value of $f_c$ may be found both together from *Fig.* 34A, B, or C by calculating $R$ from the formula $R = \frac{M}{bd^2}$ and using for $b$ the value of $(h + 2d)$. This is an alternative method to that employing equation (57). If equation (57) is used then $f_c$ should be checked either by finding $R$ or by any of the methods suggested by the formulæ given on p. 40.

(5) There yet remains to be calculated the bond stress in the bars at a section below the column face. The shear at this section is the punching shear $s$ referred to on p. 105, and the total shear on the one face is thus

$$V_1 = h \times d \times s \quad . \quad . \quad . \quad . \quad . \quad (58)$$

The unit bond stress is

$$u = \frac{V_1}{jd\Sigma o} \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where $\Sigma o$ is the sum of the perimeters of all the bars provided in the width $(h + 2d)$. The bond stress is often high and it is generally necessary to provide bars of relatively small diameter and close spacing. It will frequently be necessary also to anchor the bars by providing hooks at the ends, but this will not be necessary in the very few cases where the calculated bond stress is low.

The reinforcing bars, which can be assembled and wired together at intersections to form a mat and then propped in position before the concrete is placed, should be embedded well in from the bottom face—2 in. or 3 in. is not too much: the footing concrete is often placed under conditions far from ideal, and subsequent inspection is out of the question, so that every precaution should be taken to make a good job.

The footing may, if large, be designed with a sloping or stepped-down top, in which case care should be taken that the correct effective depth is used in the calculation of shear (diagonal tension).

Where the normal condition of a column is such that it sustains eccentric loading the footing should be placed eccentrically to the column so that the calculated bearing pressure is uniform: thus for the column investigated in the example on p. 97 a footing having a 4-in. eccentricity might be provided. Eccentricity will, of course, have less effect in varying the pressure below a footing than in a column as the ratio of eccentricity to width of section is greater in the case of the column. When conditions are such that a column is required to carry eccentric loading at irregular times a concentric footing would be provided, but the bearing pressures should be examined for all conditions of loading, and the design should be made for the maximum possible intensities. The method of determining the bearing pressures under eccentric loading is explained on p. 112.

The assumption is made in design that the distribution of pressure below a footing carrying a concentric load is uniform. Actually this is not the case. The pressure is more intense immediately below the centre of the footing and becomes less towards the sides. In a perfectly cohesionless soil the pressure distribution is parabolic, but in cohesive soils (and in practice almost all soils have some cohesion) the pressure is more nearly uniform: this is because the slight settlement which accompanies the loading of all soils results in a friction set up by the shearing action in a vertical section below the perimeter of the footing, and the soil outside the immediate footing area makes some contribution towards the support below the edges of the footing. This will be followed more easily after a perusal of the following chapter.

It is impossible to give a table of safe bearing pressures which can be applied in a mechanical way to all soils, and the following table should be considered as no more than a rough guide.

TABLE VII.

ALLOWABLE BEARING PRESSURES.

| Foundation bed. | Tons per square foot. |
| --- | --- |
| Moist sand, soft clay | 1 This may be increased if ground is retained laterally by sheet piling. |
| Dry sand and clay mixed | 2 |
| Firm dry sand, firm clay | 2–3 |
| Firm gravel, coarse hard and compact sand, very hard clay, firm chalk | 4–5 |
| Shale in level beds, hard-pan | 6–8 |
| Rock | From 8 upwards to almost any amount depending upon hardness. |

## Example of Column Footing Design.

Design a reinforced concrete footing for the column which was designed in the example on p. 95. The bearing capacity of the ground may be taken as 1 ton per square foot.

(1) BEARING PRESSURE.—

| | lb. per square foot. |
| --- | --- |
| Allowable ground pressure 1 ton | = 2,240 |
| Deduct for footing assumed 12 in. thick | = 150 |
| Net available bearing | = 2,090 |

Load to be supported = 57,250 lb. (see p. 95).

$$\text{Area required} = \frac{57{,}250}{2{,}090} = 27{\cdot}4 \text{ sq. ft.} = (5{\cdot}23)^2.$$

Using a 5-ft. 3-in. square footing, the ground pressure is

$$p = \frac{57{,}250}{(5\tfrac{1}{4})^2} = 2{,}080 \text{ lb. per square foot.}$$

(2) PUNCHING SHEAR.—Taking the allowable compressive stress as 750 lb. per square inch, the allowable punching shear $s$ is $0{\cdot}3 \times 750 = 225$ lb. per square inch.

$$\text{Then } 225 = \frac{2{,}080(5\tfrac{1}{4}^2 - 1)}{4 \times 12 \times d}$$

$$\text{and} \qquad d = \frac{2{,}080 \times 26{\cdot}5}{225 \times 4 \times 12} = 5{\cdot}1 \text{ in. (minimum effective depth).}$$

Try an 8-in. thick footing with 2-in. cover to the lower layer of bars forming the mesh. Assuming these bars to be $\frac{1}{2}$ in. in diameter, the embedment of the lower bars (to the centre of the bar) is $2\frac{1}{4}$ in., and that of the upper bars crossing at right angles is $2\frac{3}{4}$ in., so that the effective depth should be taken as $5\frac{1}{4}$ in.

$$(h + 2d) = 12 + 10\tfrac{1}{2} = 22\tfrac{1}{2} \text{ in.} = 1{\cdot}88 \text{ ft.}$$

(3) SHEAR (DIAGONAL TENSION).—

$$V = \frac{2{,}080[5\frac{1}{4}^2 - (1\cdot88)^2]}{4 \times 1\cdot88} = 6{,}640 \text{ lb. per foot width}$$

then $v = \dfrac{6{,}640}{12 \times 0\cdot9 \times 5\frac{1}{4}} = 117$ lb. per square inch. This is too high, and the effective depth should be increased. If $(h + 2d)$ did not change at the same time, the new value of $d$ could be determined directly as $\dfrac{117}{75} \times 5\frac{1}{4}(= 8\cdot2)$, but actually the new requirement will be somewhat less than this: try a $2\frac{1}{2}$-in. increase, making $d = 7\frac{3}{4}$ in. and $(h + 2d) = 2\cdot29$ ft.

The revised value of $V$ is $\dfrac{2{,}080[5\frac{1}{4}^2 - (2\cdot29)^2]}{4 \times 1\cdot88} = 6{,}170$ lb. per foot width,

and $v = \dfrac{6{,}170}{12 \times 0\cdot9 \times 7\frac{3}{4}} = 74$ lb. per square inch. This is reasonable. The punching shear will now be less than that previously calculated. The footing thickness is now 10 in.

(4) BENDING MOMENT.—The "effective width" may be taken as 2·29 ft. and $a = 2\cdot125$ ft.

$$\therefore M = 2{,}080 \times (2\cdot125)^2\left[\frac{5\cdot25}{2} - \frac{2\cdot125}{3}\right] = 18{,}000 \text{ ft. lb.};$$

$$R = \frac{18{,}000}{2\cdot29 \times (7\frac{3}{4})^2} = 131\,;\ f_c = 745 \text{ lb. per square inch};\ p = 0\cdot0095;$$

$$A_s = 12 \times 7\tfrac{3}{4} \times 0\cdot0095 = 0\cdot882 \text{ sq. in. per foot of width.}$$

($\frac{3}{4}$-in. bars at 6-in. centres = 0·88 sq. in., but on account of bond it will be preferable to use $\frac{5}{8}$-in. bars. It is unnecessary to revise $d$, but the 2-in. cover to the bars may be reduced to $1\frac{3}{4}$ in. These bars spaced at 4-in. centres are equivalent to 0·92 sq. in., and $\Sigma o = 5\cdot89$.)

(5) BOND STRESS.—The revised punching shear is

$$s = \frac{2{,}080(5\frac{1}{4}^2 - 1)}{4 \times 12 \times 7\frac{3}{4}} = 148 \text{ lb. per square inch.}$$

$$\therefore V_1 = 12 \times 7\tfrac{3}{4} \times 148 = 13{,}800 \text{ lb. per side of footing.}$$

$$\Sigma o \text{ for the width } (h + 2d) = 5\cdot89 \times 2\cdot29 = 13\cdot5$$

$$u = \frac{13{,}800}{13\cdot5 \times 0\cdot9 \times 7\frac{3}{4}} = 147 \text{ lb. per square inch.}$$

This bond stress is high and the ends of the bars should be hooked.

The $\frac{5}{8}$-in. bars would be provided at 4-in. spacing within the width $(h + 2d)$, and outside this strip a 6-in. or 8-in. spacing would be suitable. In construction these bars would be wired together into a rigid mat before being lowered into the hole excavated for the footing.

# CHAPTER XI

# EARTH PRESSURE

## Bearing Pressure below Footings.

THE example at the end of the last chapter followed the usual practice in assuming (for concentric loading) a uniform bearing below the footing slab. In actual foundations there are two factors operating against this distribution, which, although disregarded in our calculations, should yet be understood.

Consider first the case of a rigid footing not thin enough to be flexible. If this is placed on a purely granular and non-cohesive soil such as clean sand containing no particles of clay, the distribution of bearing pressure below the footing is approximately as shown in *Fig.* 73, having a value of zero at the edge.

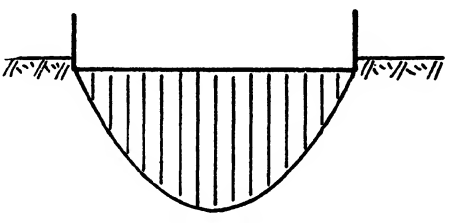


**Fig. 73.**

In a cohesive soil, however, such as clay, the surrounding body of earth bears up the material immediately below the footing by means of friction acting over the vertical planes immediately below the footing perimeter, and the bearing is more nearly uniform (*Fig.* 74).

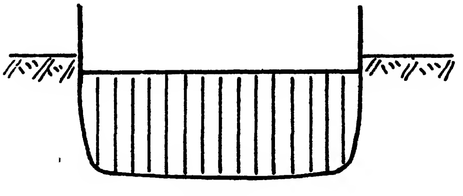


**Fig. 74.**

Consider next the case of a relatively thin and flexible footing such as either of those shown in *Fig.* 75. The bearing resistance in the earth is developed by a slight amount of settlement on account of the stress-strain relation. At

the same time the footing slab is taking up the load and, being flexible relative to the type previously considered, it bends as a beam or cantilever. It therefore follows that more settlement of the earth, or compression under load shall we say, occurs under the more rigid portions of the footing immediately below the column or wall whose load is to be transferred. The result is that the bearing pressure is more intense in these regions, and the pressure, even in a cohesive soil, tends to vary in the manner indicated in *Fig.* 75. (*Figs.* 73, 74, and 75 are diagrammatic only.) There is one type of soil where this will not occur; if the foundation bed is of soft clay containing much water the plasticity will tend to make the pressure distribution uniform—the initial settlements will be

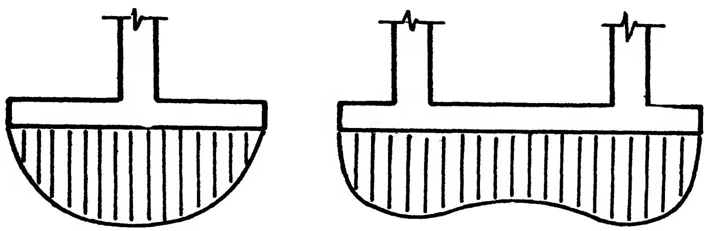


**Fig. 75.**

plastic rather than elastic, and the pressure will more nearly resemble hydrostatic pressure, uniform over all points at the same depth. The most unequal distribution will occur in rock where there is no plastic deformation.

Variations of bearing pressure such as those mentioned tend to reduce the loading on the flexible parts of the footing, and the assumption of a uniform distribution is therefore on the side of safety.

In the foregoing discussion it has been assumed that the loading brought to the footing is concentric or symmetrical so that the resultant of the bearing pressure passes through the centre of the footing area. Where the loading is eccentric, as for instance in all cases where a retaining wall supports a side load, the resultant pressure on the foundation bed must meet and oppose the resultant force of the applied loads. The method of obtaining the resultant of the applied forces is demonstrated in the following chapter, but for the present we can assume a resultant force such as $R$ shown in *Fig.* 76, making an angle $\theta$ with the horizontal footing base which it intersects at $o$. $R$ can be resolved into two forces, $R \cos \theta = H$ acting horizontally, and $R \sin \theta = P$ acting vertically. The first of these, $H$, must be resisted by friction between the slab and the foundation bed, while the second must be resisted by vertical pressure on the foundation bed. Such pressure is assumed to vary uniformly as represented by $DEFG$ in *Fig.* 76. This area representing the intensity of pressure may be split up into a rectangle of pressure $DEHG$ whose resultant acting through the centre of gravity is at the mid-point, and a triangle of pressure whose resultant is one-third of $b$ from the side. The resultant of these two resultants will be equal and opposite to $P$, and will have the same point of application in the base.

The distance $co$, where $c$ is the middle point of $AB$, is termed the eccentricity of the resultant force, and is denoted by $e$. Since $R$, and therefore $P$ and can be determined easily from the applied forces it is possible to write do-

equations * connecting these values to the pressure rectangle and triangle, so that the values of the bearing pressure may be determined. This has been done in *Fig.* 77 for four possible conditions.

The simplest condition is Case 1 where the loading is concentric and therefore $e = 0$ and the bearing is uniform. Case 2 occurs when $e$ lies between 0 and $\frac{b}{6}$. Case 3 is the limiting condition of Case 2 when $e = \frac{b}{6}$ and the pressure diagram is triangular. Case 4 occurs when $e$ exceeds $\frac{b}{6}$, and, since tension cannot be

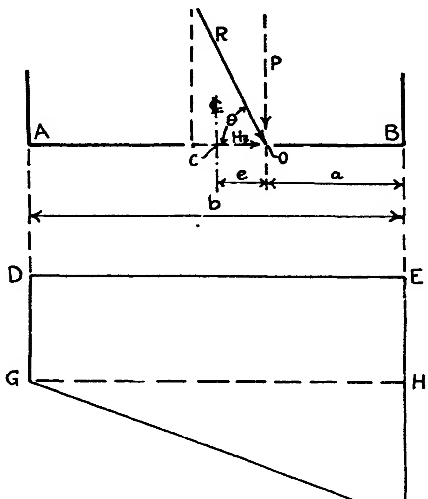



Fig. 76.

resisted between base and ground, pressure occurs over a shortened base. The formulæ for solving these cases are developed in *Fig.* 77. In Case 1 the bearing pressure intensity is obviously

$$p = \frac{P}{b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Case 2 is solved by

$$\frac{p_A}{p_B} = \frac{P}{b}\left[1 \pm \frac{6e}{b}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

Case 3 by

$$p_A = 0, \text{ and } p_B = \frac{2P}{b} \quad . \quad . \quad . \quad . \quad . \quad (62)$$

and Case 4 by

$$p_B = \frac{2P}{3a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

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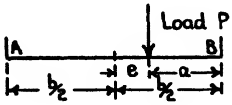
* These equations are obtained by resolving vertically and by taking moments about the footing.

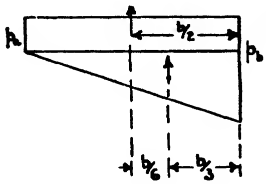
## DISTRIBUTION OF BEARING PRESSURE.

Assumption: Uniform variation of pressure intensity.




Load P = Vertical component of forces acting on the wall.
(Horizontal component taken by friction.)


Case 1 – Uniform bearing when $e = 0$.




Case 2 – P cuts the base inside the middle third
e is less than $b/6$.

Resolving Vertically

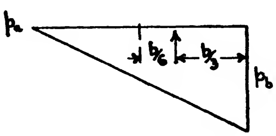
$$\frac{p_b + p_a}{2} \times b = P \qquad \therefore p_b + p_a = 2P/b$$

Taking Moments about ℄ of base

$$(p_b - p_a)\,\frac{b}{2} \times \frac{b}{6} = P.e. \qquad \therefore p_b - p_a = \frac{2P.e. \times 6}{b^2}$$

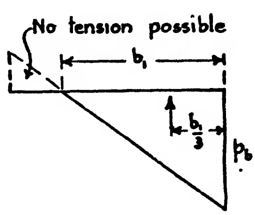
$$\therefore 2p_b = \frac{2P}{b} + \frac{2P.e.6}{b^2} = 2\frac{P}{b}\left[1 + \frac{6e}{b}\right]$$

and $p_a = \dfrac{P}{b}\left[1 - \dfrac{6e}{b}\right]$




Case 3 – Limiting condition for compression over whole of base. P cuts base at edge of middle third. $e = b/6$

$$p_b \times b/2 = P \qquad \therefore p_b = \frac{2P}{b} = \text{twice average.}$$




Case 4 – Shortened 'base'. P cuts base outside the middle third. e exceeds $b/6$.

$$b_1 = 3a = 3(b/2 - e)$$

$$p_b \cdot \frac{b_1}{2} = P \qquad \therefore p_b = \frac{2P}{b_1}$$

Fig. 77.

In each of these equations the calculations are made for a strip of footing 1 ft. wide. If a square or rectangular footing is dealt with as a complete unit, instead of selecting a strip 1 ft. wide, the value of $P$ used in the equation should be the total load divided by the length of footing which supports it. It is this same method illustrated in *Fig.* 77 which is used to determine the stresses in a homogeneous section as suggested on p. 97. For this purpose equation (61) will serve all conditions if tension can be resisted in the section. A positive result will represent compression, and a negative result tension.

In actual conditions the pressures operating will not be perfectly uniform, but will be modified in the manner suggested at the beginning of this chapter to an extent depending upon the cohesive properties of the material comprising the foundation bed, and the flexibility of the footing. The resultant forces, both applied and reacting, must however balance. The variations in distribution from the ideal conditions assumed may, as previously explained, be neglected as they are on the side of safety.

### Side Pressure.

Consider the forces acting in the ground below a footing such as is shown in *Fig.* 78. If the ground to the left of $AD$ or to the right of $BC$ were removed—

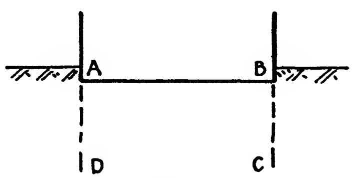



**Fig. 78.**

as would be the case if deep trenches were dug—the material composing $ABCD$ would burst out sideways. The material which was removed had previously been exercising a buttressing effect, and horizontal pressure existed across the planes $AD$ and $BC$. The same would be true whatever the nature of the load above $AB$, and it is true when no superload is applied, for the earth itself forming $ABCD$ has weight and develops a sideway thrust.

Earth faces are seen standing vertically at the sides of sand-pits, although local vibrations or disturbances are likely to cause minor collapses, and rock faces stand sheer for great heights. Such stability is due to cohesion, and the power to stand up without exerting sideways pressure is due to the possession of two properties, namely, cohesion and internal friction. Water and other liquids relatively speaking possess neither of these qualities: they flow sideways unless restrained, and exert pressures in all directions equal in intensity to the weight of fluid in the full depth to the point under consideration; we refer to this force as the pressure head. Between the vertically-standing (or even overhanging) rock face and the free-flowing fluid there are numberless conditions and states. The property which allows rock to overhang is its cohesion. This factor is, apart from special circumstances which are judged only by experience,

ruled out in the consideration of earth pressures, and design is carried out by consideration of the effects of internal friction. This friction between the grains of a material permits it to pile up into a heap instead of flowing like water, as would be the case were all the grains polished and frictionless: thus a heap of dry loose sand free from cohesion will stand up at an angle of say 35 deg. The angle, designated $\phi$, which this free surface makes with the horizontal is termed the angle of repose, and it is to all intents and purposes equal to the angle of internal friction, the tangent of which ($\tan \phi$) is called $\mu$ or the coefficient of friction. (The student is referred here to his studies in dynamics. The static friction is slightly greater than sliding friction, but for our purposes they may be considered to be equal.)

This property of internal friction, or the ability to resist flow, controls the sideways pressure, generally referred to as lateral thrust, exerted by a granular material free from cohesion. Consider a condition $h$ ft. below ground level such as that illustrated in *Fig.* 79, but where $AB$ is of indefinite extent so that

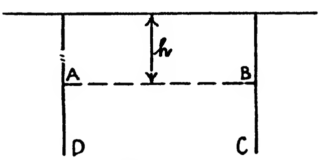



Fig. 79.

the effect of conditions below $B$ on conditions below $A$ is nil, and further consider the material immediately to the left of $AD$ to be replaced by a plate which is capable of resisting any forces as necessary; we will refer to the plate as $AD$. Now $AB$ is loaded by the earth above this level, and if the earth weighs $w$ lb. per cubic foot the vertical pressure due to this overburden is $wh$ lb. per square foot. This exerts a lateral thrust on $AD$, and it is with this that we are concerned.

There are numerous theories of earth pressure, but the best known and perhaps the most practicable is Rankine's. He considers two limiting conditions. In the first the vertical pressure of intensity $wh$ per unit of area produces a lateral thrust on $AD$ of intensity equal to $wh \times \dfrac{1 - \sin\phi}{1 + \sin\phi}$ per unit of area, where $\phi$ is the angle of internal friction already referred to. This lateral thrust is termed the active thrust, as it operates actively on $AD$ which merely passively resists it. The second condition is that in which the plate $AD$ is moved forcibly against the earth $ABCD$ in such a manner as eventually to cause $AB$ to rise. At the critical stage when movement is impending the applied lateral thrust is equal to $wh \times \dfrac{1 + \sin\phi}{1 - \sin\phi}$ and this is the limiting value of what is termed the passive thrust. Passive thrust can be developed only when a wall is moved forcibly against an earth face (as for instance by the expansion of a structure buried in the earth or by a building sliding down-hill); and the pressure then developed may be very high. If the angle of internal friction is 30 deg.,

$\sin\phi = \frac{1}{2}$, $\frac{1 - \sin\phi}{1 + \sin\phi} = \frac{1}{3}$, and $\frac{1 + \sin\phi}{1 - \sin\phi} = 3$, so that the limiting value of passive friction is in that case nine times the active pressure. $wh \times \frac{1 - \sin\phi}{1 + \sin\phi}$ is the limiting value of active pressure, and cohesion will reduce it. Passive pressure, if developed, may lie anywhere between $wh \times \frac{1 - \sin\phi}{1 + \sin\phi}$ and $wh \times \frac{1 + \sin\phi}{1 - \sin\phi}$, and cohesion could increase its high limit. In ordinary cases we concern ourselves only with active pressure, and we disregard cohesion which even in cohesive soils can be destroyed by excavating and backfilling, vibration, or water.

Let us examine this formula for active pressure. If $\phi$ has been determined, $\frac{1 - \sin\phi}{1 + \sin\phi}$ becomes a constant; $w$ (the weight of the earth) also is a constant, so that the only variable is $h$. The lateral thrust therefore is directly proportional to the depth, and the horizontal pressure diagram is therefore a triangle such as in *Fig.* 80 in which the horizontal ordinate is always proportional to $h$.

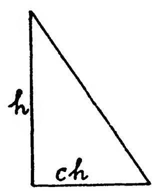



**Fig. 80.**

The values of $w$ and $\phi$ vary with the nature of the soil, but commonly assumed values are 110 lb. per cubic foot, and 35 deg. respectively, and with these figures the expression for lateral thrust intensity reduces to approximately $30h$. This horizontal pressure is therefore equal to that which would be exerted by a fluid weighing 30 lb. per cubic foot, and it is commonly referred to as the equivalent fluid pressure.

So far we have considered the horizontal pressure exerted on a wall due to non-cohesive earth or any granular material filled up behind it to a level surface. The pressure is triangular, and the centre of pressure on the wall is at the same level as the centre of gravity of the triangle, one-third of the depth measured up from the base.

There are two other conditions to consider:

(1) Surcharge due to a sloping bank, and
(2) Live load surcharge.

Surcharge due to a sloping bank is shown in *Fig.* 81. If the slope is at the angle of repose $\phi$ and is of such proportions as to be considered unlimited in extent, Rankine's theory assumes the unit pressure at any point at depth $h$ in the vertical plane of the wall to be $p = wh \times \cos\phi$ and to act in a direction parallel to the surface.

In practice the slope is of limited extent, and is generally at an angle less than the real angle of repose. A simple and practicable method is to assume

a pressure diagram as shown in *Fig.* 82. The apex of the pressure triangle is taken at a level half-way up the surcharge, and the pressure on the wall is considered in the two simple parts as a rectangle (*BDEF*) and a triangle (*CFE*).

The live load surcharge is treated in a similar manner by assuming a uniformly distributed load to be replaced by an equivalent depth of earth sur-

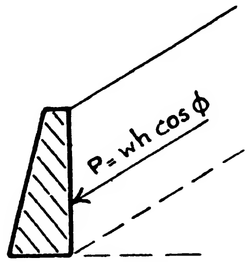



**Fig. 81.**

charging the wall; the apex of the triangle is taken at the level of the top of the equivalent earth filling, and the pressure diagram is dealt with as already described. With a concentrated live load some reasonable method of dispersion should be assumed or an "equivalent distributed load" adopted.

The term centre of pressure has been used to refer to the position of the resultant of the horizontal pressures. The position of this is confined within

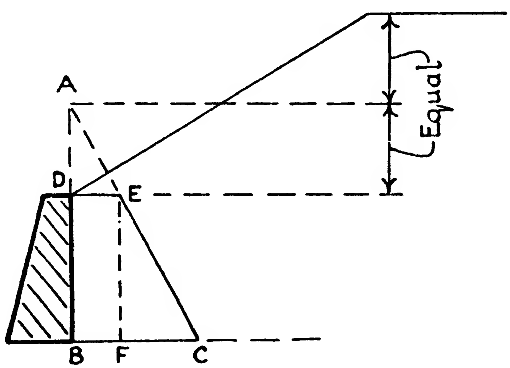



**Fig. 82.**

narrow limits. In a triangular pressure diagram such as that shown in *Fig.* 80 the centre of pressure is at a distance $\frac{h}{3}$ above the base. With a rectangle the corresponding distance is $\frac{h}{2}$, and therefore with a pressure diagram such as that shown in *Fig.* 82, the value must lie between $\frac{h}{2}$ and $\frac{h}{3}$; the possible range is therefore somewhat less than $\frac{h}{6}$.

The preceding discussion is theoretical, but is nevertheless sufficiently accurate for design purposes, and the methods described are in wide general use. Other more complicated methods also are in use, but it is doubtful if the results are any more accurate.

In practice the earth may tend to arch behind the wall and load the top more than the bottom so that the centre of pressure is more than $\frac{h}{2}$ from the bottom, but the total pressure will probably be less than the theoretical pressure and the calculations will be on the safe side.

On the other hand, cohesion in the earth tends to relieve the pressure near the top of the wall. Cohesion is so variable and can scarcely be relied upon in ordinary soils. Much of this property is destroyed when the ground is disturbed in excavating and backfilling, and it is affected by water content. Water in quantity will also affect the value of $\phi$ (angle of repose) and therefore the pressures, besides exerting its own static fluid pressure, and for this reason, where its presence is undesirable, all necessary measures should be taken for drainage of the subsoil. Clay is the most troublesome material to deal with. It absorbs moisture and expands, it tends to flow under pressure, and its nature generally is most changeable. Firm clays often make a reasonably satisfactory foundation bed, but clay should not be used in filling or embankment where the properties referred to are likely to cause trouble. Filling should be placed in level and shallow layers, and compacted by rolling or puddling.

# CHAPTER XII

## DESIGN OF SIMPLE RETAINING WALLS

RETAINING walls are simple structures, but are, nevertheless, found frequently in practice to give trouble. Often this is because one or two fundamental rules have been disregarded either in design or in construction. These points are elementary and yet so important that they will be referred to at length after the details of design have been explained.

The function of a retaining wall is to retain a material, generally earth, so as to prevent it from taking up its angle of repose. Thus in *Fig.* 83 a retaining

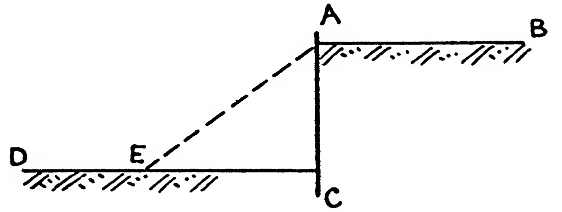



**Fig. 83.**

wall would be constructed in position $AC$ to prevent the earth at the high level $AB$ from extending to $E$ bounded by the slope $AE$, so leaving $DC$ clear at the lower level, perhaps for a road. Or again $AC$ may be constructed in a yard to provide for filling coal or coke up to the level $AB$ while confining it to a limited area. In both cases there will be, as we saw in the previous chapter, an active side pressure on the wall at any point depth $h$ below the top of the material (of weight $w$ lb. per cubic foot) equal in intensity to $wh \times \dfrac{1 - \sin\phi}{1 + \sin\phi}$ in lb. per square foot, where $\phi$ is the angle of internal friction and is taken equal to the angle of repose.

This pressure affects the wall in two ways. There is a tendency for the wall (1) to slide away from the retained material, and (2) to overturn by rotating about the base $C$. These two can of course both happen, the wall commencing to slide, then being checked at the base and overturning.

### Mass Concrete Retaining Wall.

The simplest form of wall is a massive block of concrete or masonry such as that shown in *Fig.* 84. The active forces are the horizontal pressure $H$ from

the retained earth, and $W$ the weight of the wall itself. The passive forces resisting the tendencies referred to as (1) and (2) are (*a*) friction (horizontal) in the foundation bed, preventing sliding, and (*b*) vertical bearing pressure, similar to one of the conditions shown in *Fig.* 77. The distribution of this pressure provides a moment resisting the overturning effect.

The method of determining these is demonstrated by a simple example below.

EXAMPLE.—Design a mass concrete wall 10 ft. high of rectangular section to retain an earth fill. Take the equivalent fluid pressure (see p. 116) as 30 lb. per square foot and neglect surcharge.

A 1-ft. slice of wall is taken for investigation. It is necessary to assume from experience the width of the base: this width will decide which of the conditions shown in *Fig.* 77 will prevail. In practice the width is taken generally between a quarter and one-half the height. In the present example we shall assume a width of base equal to 4 ft.

The triangle of pressure (horizontal) will have a value at its base of 30 × 10, or 300 lb. per square foot, and its area, which represents the total active horizontal pressure, is 1,500 lb.

The weight of the wall is $4 \times 10 \times 150 = 6{,}000$ lb.

The coefficient of friction necessary to prevent the wall from sliding, neglecting any resistance offered by material in front of the toe, is thus $\frac{1{,}500}{6{,}000}$ or 0·25. In practice the coefficient of friction varies from about 0·2 for wet clay to about 0·6 for good sand or gravel.

The next step is to examine the bearing pressure so as to determine the factors of safety against overturning and settlement. The simplest method is to take moments about the heel $B$, setting down the calculations as shown below.

TABLE OF LOADS AND MOMENTS.

| | Vertical load. | Lever arm | Moment $B$. |
|---|---|---|---|
| Wall . . . . | 6,000 | 2 | 12,000 |
| $H_e$ = 1,500 lb. . . | — | $\frac{10}{3}$ | 5,000 |
| | 6,000 lb. | 2·833 ft | 17,000 ft. lb. |

$$\frac{b}{2} = 2{\cdot}0$$

$$\therefore e = 9{\cdot}833 \text{ ft.}$$

This shows that the resultant cuts the base just forward of the edge (that is, outside) of the middle-third, corresponding to Case 4 of the conditions represented in *Fig.* 77. Using the appropriate formula,

$$a = 1{\cdot}167 \text{ and } b_1 = 3 \times 1{\cdot}167 = 3{\cdot}5 \text{ ft.}$$

$$p_B = \frac{2 \times 6{,}000}{3{\cdot}5} = 3{,}430 \text{ lb. per square foot.}$$

The average pressure over the bearing area is one-half this value, or 1,715 lb. per square foot.

The wall will not overturn unless the resultant falls outside the base; this happens when $e$ exceeds $\frac{b}{2}$, but it is obviously undesirable that such a condition should be even approached. The term factor of safety against overturning has been referred to; it is not really necessary to find it as the information given by the preceding calculations is sufficient to determine the suitability of the design, but it is as well that the reader should understand the meaning of the expression. Overturning, if it occurs, takes place by rotation about the forward edge $A$. The overturning moment is that due to $H_e$, the value of which moment in the example given is 5,000 ft. lb. The stabilising moment is provided by the weight of the wall (6,000 lb.) acting at a distance of 2 ft. away from $A$. The factor of safety against overturning is given by the ratio of the second of these moments taken about $A$, to the first, or $\frac{12{,}000}{5{,}000} = 2{\cdot}4$.

Now let us consider this wall a little more closely. It is obvious that the design is wasteful. Since the object is to get the resultant to pass inside the middle-third of the base, the body of material in the front third of the wall (apart from the base) is operating against this object—it provides a clockwise moment about $A$ (*Fig.* 84) but an anti-clockwise moment about the forward edge of the

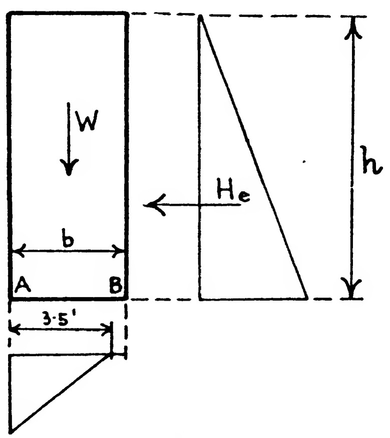



**Fig. 84.**

middle-third; it is therefore not of service until the wall is getting near the point of overturning. It also adds to the total weight to be carried on the foundation bed. Our first improvement is therefore to cut out the main portion of the wall in front of the middle-third without reducing the width of the base. Our next improvement is to replace the main portion of the rear of the wall by earth, using the weight provided by it to supply the stabilising moment against overturning. The earth (or other material) is not so heavy as the concrete, but it is much cheaper. By this method we arrive at variations in the elementary type of rectangular wall, as shown in *Fig.* 85. Type (*b*) is suitable for mass

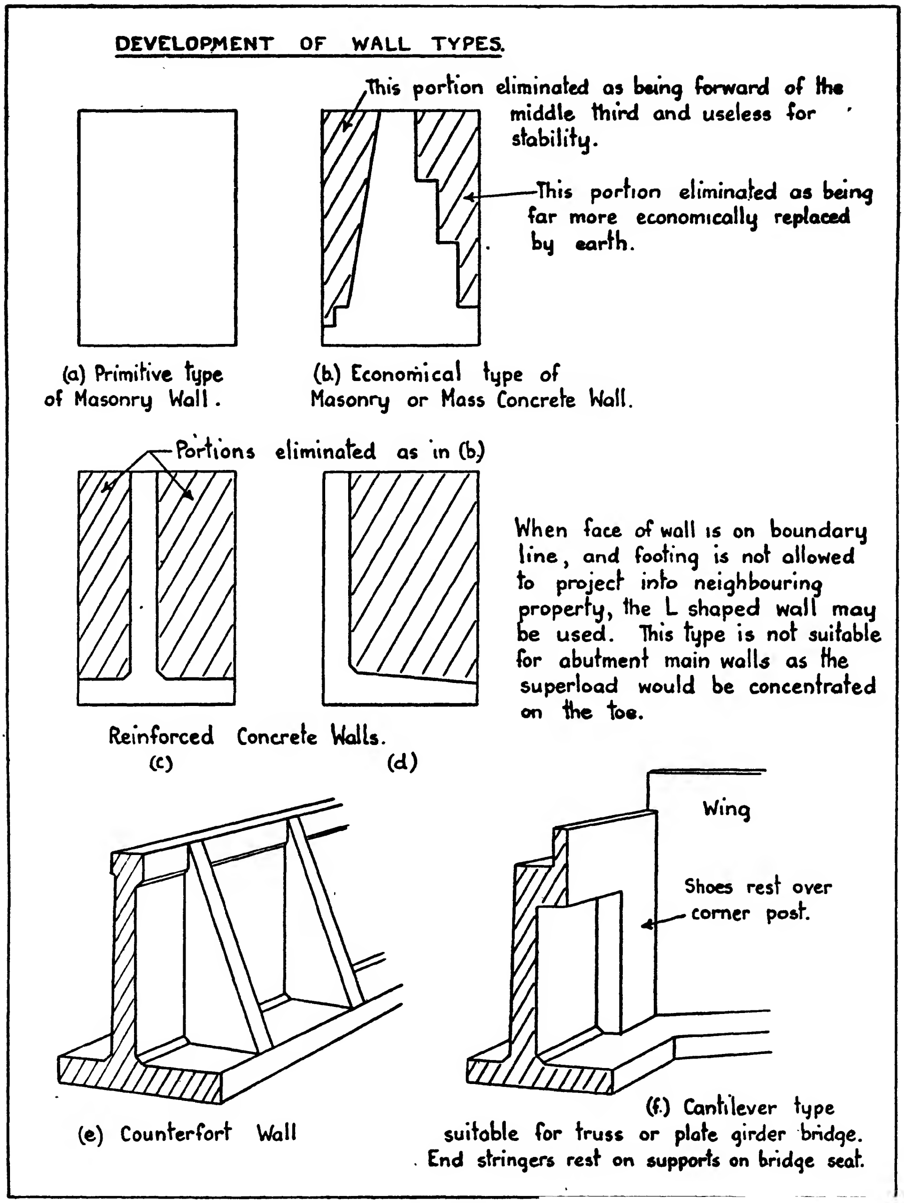

DEVELOPMENT OF WALL TYPES.
This portion eliminated as being forward of the middle third and useless for stability.
This portion eliminated as being far more economically replaced by earth.
(a) Primitive type of Masonry Wall.
(b) Economical type of Masonry or Mass Concrete Wall.
Portions eliminated as in (b)
When face of wall is on boundary line, and footing is not allowed to project into neighbouring property, the L shaped wall may be used. This type is not suitable for abutment main walls as the superload would be concentrated on the toe.
Reinforced Concrete Walls.
(c)
(d)
Wing
Shoes rest over corner post.
(e) Counterfort Wall
(f.) Cantilever type suitable for truss or plate girder bridge. End stringers rest on supports on bridge seat.

Fig. 85.

concrete or masonry; type (*c*) for reinforced concrete. This latter is, apart from (*b*), the commonest type of wall and will be explained fully below and then demonstrated. Type (*d*) is discussed in *Fig.* 85, and should be avoided where possible. Type (*e*) is used for very large walls, and although the detailed design is not difficult it is a little more complicated than the ordinary cantilever wall (type *c*). Type (*e*) will be dealt with later.

### The Vertical Cantilever Retaining Wall.

Let us now consider type (*c*). The design consists of three parts, the wall, the toe, and the heel.

The wall is designed as a vertical cantilever, fixed by the footing, and loaded by the horizontal pressure from the earth. The bending moment is determined at various sections and the necessary amount of reinforcement provided. Shear in these walls is negligible.

The forces acting on the footing to provide the counter-moment for the wall are the weight of the wall itself and of the earth above the heel and the bearing pressure below the whole footing.

The toe is designed by considering the upward bending caused by this bearing pressure, together with the shear and the bond stresses.

The heel is designed by considering the downward bending from the earth load, relieved by the upward pressure from bearing, together with shear and bond.

In both of these parts of the footing the weight of the concrete in the footing, as affecting the moments and shear, is considered. The moments, and corresponding bond stresses, are taken in the footing immediately below the wall faces; the shear in the heel is taken at the wall face where cracking would appear, but in the toe it is taken at the section where the 45-deg. plane from the wall face intersects the bottom steel, in the same manner, and for the same reason, as it is taken in the column footing (see p. 105).

The method is made quite clear by the following example. As in the case of the mass wall reasonable proportions must be assumed and the concrete stresses investigated; at the same time the necessary amount of reinforcing steel is determined as shown.

**Example.**—Design a vertical cantilever retaining wall of 15 ft. overall height. The equivalent fluid pressure may be taken as 30 lb. per square foot, and the bearing pressure should not exceed 1 ton per square foot. The allowable stresses are, 16,000 lb. per square inch in the steel and 750 lb. per square inch in the concrete.

THE WALL.—For a 30-lb. per square foot fluid pressure the required width of footing is approximately 0·45 of the overall height, or $15 \times 0{\cdot}45 = 6{\cdot}75$ ft. Using this footing width, and placing the wall face at the forward edge of the middle-third we get a trial section such as that shown in *Fig.* 86. In practice the front face of the wall would be battered, but it is here taken as vertical so as to keep the calculations simple; in this way the student can concentrate on the method. A 12-in. minimum thickness has been employed, so that with 2-in. embedment $d = 10$ in.

The pressure intensity on the wall at any depth $h$ is $30h$, and the area of

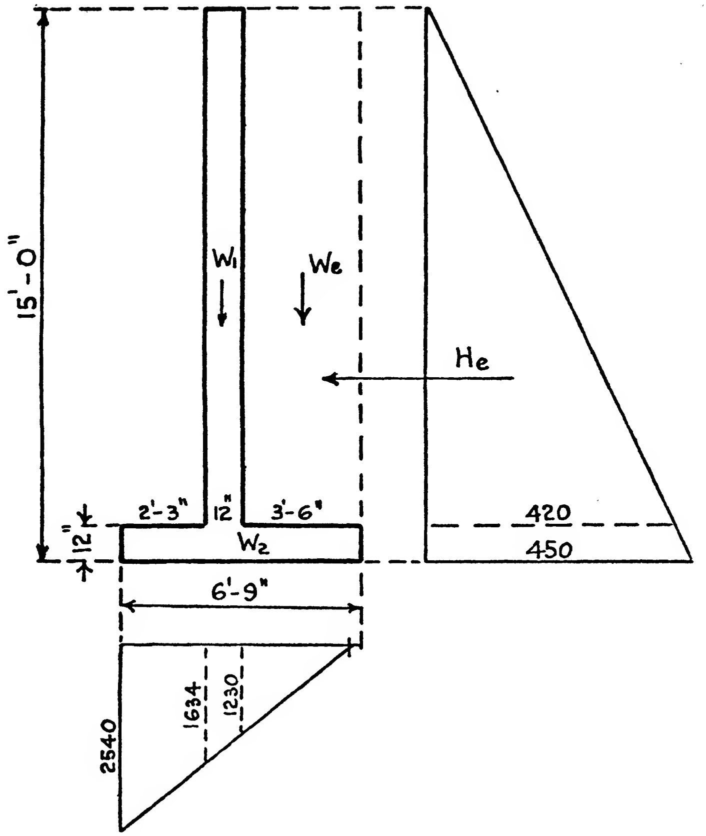



Fig. 86.

the pressure diagram down to that level is therefore $30h \times \frac{h}{2} = 15h^2$. The centre of pressure is $\frac{h}{3}$ above the section considered, so that the moment at that section is $15h^2 \times \frac{h}{3} = 5h^3$. The Table gives the moments at several depths, with $R$, $p$ and $A_s$ as described in Chapter IV.

| Section at depth $h$ (ft.). | $M = 5h^3$ (ft. lb.). | $R = \frac{M}{bd^2}$ | Steel ratio ($p$). | $A_s$ (sq. in.). |
|---|---|---|---|---|
| 3 | 135 | 1·35 | — | — |
| 6 | 1,080 | 10·8 | — | — |
| 9 | 3,645 | 36·5 | 0·00245 | 0·294 |
| 12 | 8,650 | 86·5 | 0·0061 | 0·733 |
| 14 | 13,750 | 137·5* | 0·01 | 1·20 |
| 15 | 16,880 | — | — | — |

* This indicates that the concrete stress is slightly in excess of 750 lb. per square inch.

These steel areas are plotted on squared paper as a curve showing the area required at various depths, and from this curve suitable bar sizes and spacings are selected as shown in *Fig.* 87. Where the front face of the wall is battered care should be taken to use the correct value of $d$ in calculating $R$ and $p$.

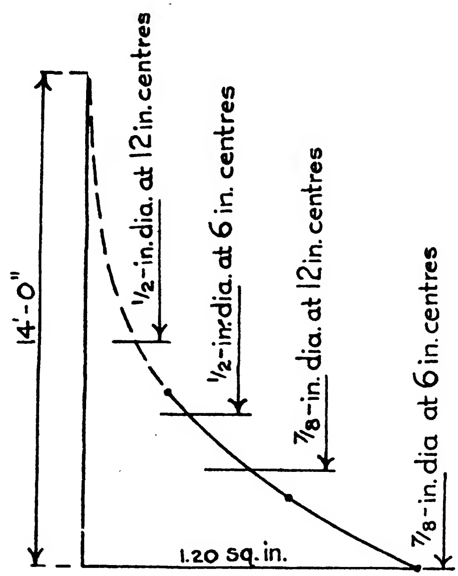



**Fig. 87.**

The bars are spliced by extending them " bond distance " below the section at which they are " required " (see *Fig.* 88). It is not really necessary to hook the bars where they are stopped off—whether they are hooked or not some local slip is almost bound to occur but it will not be serious. Distribution (or " temperature ") steel is provided by ½-in. bars spaced at 12-in. centres.

FOOTING.—$W_1$ is the weight of the wall, $W_2$ of the footing, and $W_e$ of the earth. The bearing pressures are found as previously described.

TABLE OF LOADS AND MOMENTS.

| | Loads (lb.). | Lever arm (ft.). | Moment $B$ (ft. lb.). |
|---|---|---|---|
| $W_1(14 \times 150)$ | 2,100 | 4 | 8,400 |
| $W_2(6\frac{3}{4} \times 150)$ | 1,010 | 3·375 | 3,410 |
| $W_e(3\frac{1}{2} \times 1{,}400)$ | 4,900 | 1·75 | 8,600 |
| $H_e$ | — | (See Table, p. 124.) | 16,880 |
| | 8,010 | 4·655 | 37,290 |

$$\frac{b}{2} = 3{\cdot}375 \text{ ft.}$$

$$e = 1{\cdot}28 \text{ ft.}$$

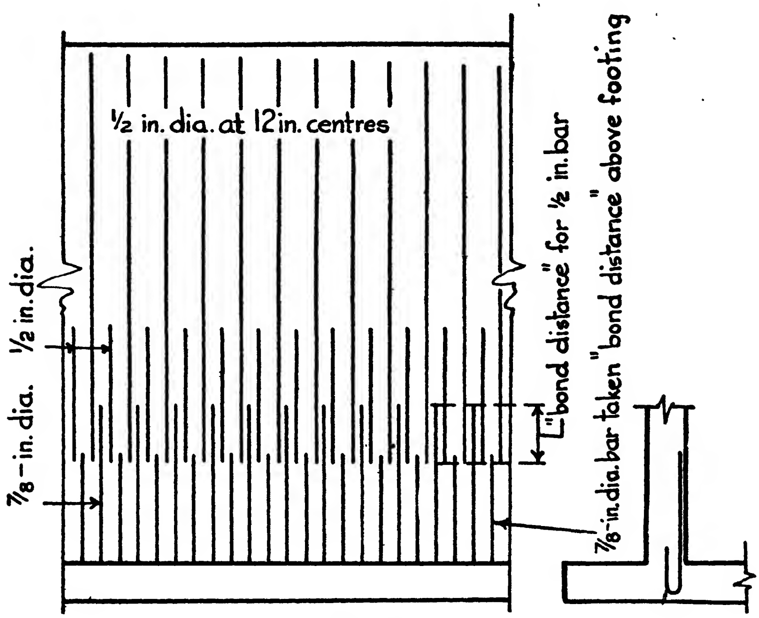



Fig. 88.

The resultant falls slightly outside the middle-third and bearing therefore occurs over $3 \times 2{\cdot}1 = 6{\cdot}3$ ft. of the base, and the pressure diagram is as shown in *Fig.* 86. The bearing pressure at the toe is $\frac{2 \times 8{,}010}{6{\cdot}3} = 2{,}540$ lb. per square foot, and below the front and rear faces of the wall is 1,634 lb. per square foot and 1,230 lb. per square foot respectively. The pressure increment is $403\frac{1}{2}$ lb. per square foot per foot.

THE TOE.—The moment is taken from the rectangle of pressure, deducting the weight of concrete in the footing, and the triangle of pressure.

$$M = (1{,}634 - 150)\,\frac{2\frac{1}{4}^2}{2} + 906 \times \frac{2\frac{1}{4}}{2} \times \frac{2}{3} \times 2\frac{1}{4}$$

$$= 3{,}750 + 1{,}530 = 5{,}280 \text{ ft. lb.}$$

$$R = \frac{5{,}280}{1 \times 10^2} = 52{\cdot}8\ ;\ p = 0{\cdot}0036\ ;\ \text{and } A_s = 0{\cdot}432 \text{ sq. in.}$$

($\frac{1}{2}$-in. bars at 5-in. centres = 0·47 sq. in. $\Sigma o = 3{\cdot}77$.)

The shear where the 45-deg. plane intersects the bottom steel, 10 in. from the wall face, is

$$V = (2{,}540 - 150)\ 1{\cdot}42 - \frac{(1{\cdot}42)^2}{2} \times 403\tfrac{1}{2} = 3{,}000 \text{ lb.}$$

$$v = \frac{3{,}000}{12 \times 0{\cdot}9 \times 10} = 27{\cdot}8 \text{ lb. per square inch.}$$

Calculation of bond stress below the face of the wall:

$$V_1 = (1{,}634 - 150)\ 2\tfrac{1}{4} + 906 \times \frac{2\tfrac{1}{4}}{2} = 3{,}340 + 1{,}020 = 4{,}360 \text{ lb.}$$

$$u = \frac{4{,}360}{3\cdot77 \times 0\cdot9 \times 10} = 128 \text{ lb. per square inch.}$$

These bars should be hooked at the ends.

THE HEEL.—The moment and shear are taken from the weight of the earth and footing acting downwards, and the pressure triangle acting upwards.

| | Shear. | Moment. |
|---|---|---|
| Earth | 4,900 | |
| Concrete $3\frac{1}{2} \times 150$ | 525 | |
| | $5{,}425 \times 1\frac{3}{4}$ | $= 9500$ |
| Deduct bearing $\dfrac{1{,}230 \times 3\cdot05}{2} =$ | $1{,}875 \times 1\cdot02 =$ | 1,910 |
| | 3,550 lb. | 7,590 ft. lb. |

Shear. $V = 3{,}550$ lb.

$$v = \frac{3{,}550}{12 \times 0\cdot9 \times 10} = 33 \text{ lb. per square inch.}$$

Moment. $M = 7{,}590$ ft. lb.

$$R = \frac{7{,}590}{10^2} = 76$$

$$p = 0\cdot0053\ ;\ A = 0\cdot636 \text{ sq. in.}$$

($\frac{5}{8}$-in. bars at 5-in. centres = 0·74 sq. in. $\Sigma o = 4\cdot71$.)

BOND STRESS.—

$$u \frac{3{,}550}{4\cdot71 \times 0\cdot9 \times 10} = 84 \text{ lb. per square inch.}$$

Back pressure from the earth in front of the toe should be neglected. It is unreliable and in any case the pressure, except in walls buried with very deep foundations, is low and acts with a very small leverage. Where the wall face is exposed to water pressure it must be designed accordingly.

*Fig.* 93 (p. 133) is provided as a design chart for reinforced concrete cantilever retaining walls designed for an equivalent fluid pressure of 30 lb. per square foot.

When piles are used below wall footings the upward reaction occurs in concentrations and the loads on the piles are found as follows.

Case 1.—Two rows of piles (*Fig.* 89). The resultant is found by the method previously described and moments are taken about the front (or rear) pile.

$$P_1(a + b) = Ra$$

$$\therefore P_1 = \frac{Ra}{(a + b)} \qquad (64)$$

$$P_2 = R - P_1 \qquad (65)$$

Case 2.—Three or more rows of piles (*Fig.* 90). The bearing pressures which would occur without piles are obtained as shown in *Fig.* 77 and the resulting

pressure diagram is split up by vertical lines midway between the pile positions; these sub-divided areas are then allotted to their respective piles.

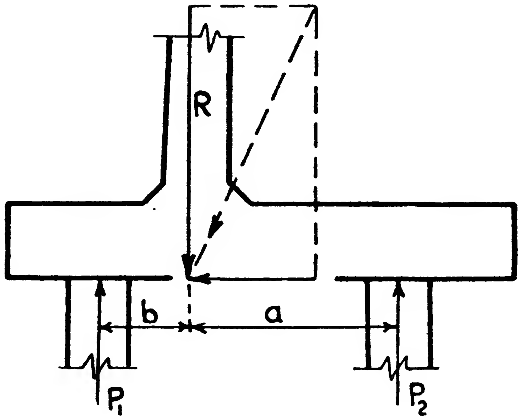



**Fig. 89.**

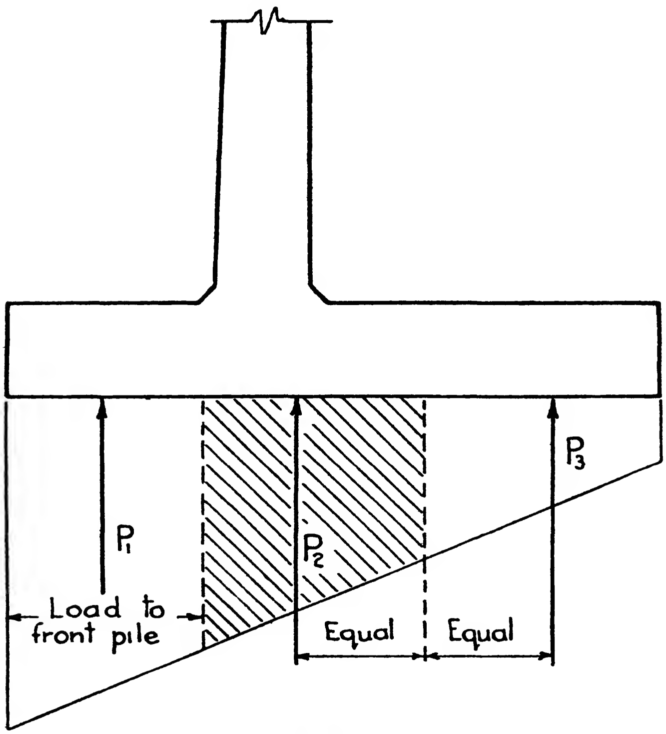



**Fig. 90.**

Since the vertical load is now taken on the piles the ground cannot be assumed to provide the necessary friction against sliding. The front row of

piles should therefore be slightly battered to provide a horizontal component, and the toe of the footing should be constructed against firm ground.

The piling will slightly affect the design of the toe and heel. The calculation is made in the same manner as described on pp. 126–7, but in place of the bearing pressure made up of a rectangle and triangle in the pressure diagram the bearing is assumed to be concentrated along the centre line of the rows of piles. If the piles are not too widely spaced longitudinally along the wall it is assumed that the distribution along this line is complete, or in other words the reaction from the piles is treated as a knife-edge thrust, and the footing designed for the average 1-ft. wide slice. It is as well in this case to provide a few reinforcing bars running longitudinally above the piles in both the top and bottom of the slab to aid the distribution.

Shear may be dealt with by the same assumptions. The maximum shear in the heel may occur either at the line of the rear piles or at the wall. In the toe the 45-deg. plane is considered as referred to on p. 123 : if this plane intersects the base of the footing between the wall face and the pile the shear should be kept within the limits previously suggested ; if, however, this plane intersects the pile at the base of the toe, or falls beyond it, shear may reasonably be neglected on the assumption that the thrust is carried up in the 45-deg. plane into the wall slab.

## General Considerations.

In detailing the reinforcing steel the arrangement of the bars should be simple so as not to add unnecessarily to the difficulties of construction. The vertical bars in the wall should, unless the wall is only a few feet in height, be spliced immediately above the footing. The bars forming the splice should not be wired together in the lap, but should be spaced out so that the concrete can bond effectively all round each bar.

Compression of the ground occurs, however slight, owing to the bearing pressure below the footing, and since the pressure below the toe is greater than that below the heel the wall will tilt slightly forward. A batter is generally provided as the wall thickness needs to be greater at lower depths to resist the increasing moment, and for the reason explained above this batter should be on the front face. A forward tilt on a " vertical " wall is immediately detected, but with a battered face the slight tilt is not apparent. The reinforcing steel is also then placed conveniently in a true vertical plane which in a cantilever wall is not the case if the rear face is battered.

Shear in the wall slab is negligible, and bond stress need not be calculated, but the reinforcing steel must be well anchored into the footing slab. In no circumstances may the reinforcing bars provided for the heel of the footing be bent up to provide wall steel as this would spall off the concrete and pull out at the re-entrant angle (see *Fig.* 91) ; the bars should cross over and anchor separately as shown by the dotted lines.

To reduce the risk of water pressure occurring behind a wall the fill should be well drained, and weep holes at reasonable intervals—say 4 or 5 ft. vertically and 10 ft. horizontally, varied according to conditions—should be provided in the walls. Glazed earthenware pipes 3 in. or 4 in. in diameter will serve this

purpose. The back of the wall, in the region of these drains, should be packed with graded gravel which acts as an open drain through which a free flow of water is afforded. No ditch or open drain should be constructed in front of the toe of the footing near or below foundation level.

Vertical construction joints must be provided in all retaining walls (except in watertight tanks) to allow for shrinkage of the concrete and for expansion and contraction with changing temperature. These joints, which are keyed as

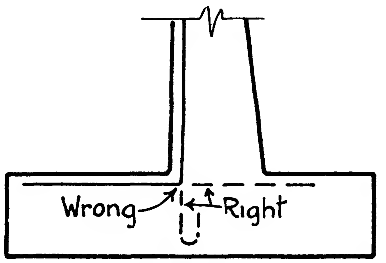



**Fig. 91.**

shown in *Fig.* 92, should be at about 30-ft. intervals or less. This applies to both reinforced concrete and mass walls. The key, which can be provided merely by nailing a temporary batten down the middle of the board forming a stop between the forms during pouring, will prevent one unit of the wall from moving in front of the other. No expansion material need be provided in the joint, but a slight groove will improve the appearance of the joint and prevent it from having the appearance of a crack.

If concrete pouring has to be discontinued at some stage between the footing and the top of the wall, the old surface must be well hacked to remove scum,

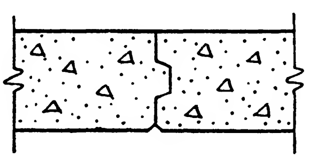


**Fig. 92.**

moistened, and then treated with a rich mortar as concreting is resumed. If this is not done a bad joint is likely to result through which seepage will occur followed by rusting of the reinforcing bars.

## The Counterfort Retaining Wall.

As was shown in *Fig.* 85, all retaining walls are based on the elementary type of wall formed of a massive rectangular block. The economies effected in the vertical cantilever retaining wall were explained on p. 121. When the height of wall necessary to retain a fill is great the wall thickness at the base becomes excessive in just the same way that a long span slab becomes excessively thick. The method suggested on p. 71 for dealing with that difficulty was to eliminate

the wasteful concrete "below" the neutral axis by boxing it out, and concentrating the reinforcing steel in the ribs so formed; the result was the T-beam. Precisely the same method is adopted in the present case, only as the "T-beam" is a cantilever (from the footing) instead of being supported at both ends, the beam (termed "counterfort") is triangular in elevation (see *Fig.* 85): the wall slab provides the flange of the T-beam. The counterfort wall is thus seen to be of the cantilever type, but whereas the "cantilever" wall is self-sustaining foot by foot, the counterfort wall is stable only in the bay units. The horizontal earth thrust on the wall is "supported" by the wall slab spanning horizontally from counterfort to counterfort; this slab is therefore designed as a slab continuous over supports and loaded by the horizontal earth thrust. Since the counterforts are constructed on the earth side of the slab they do not as "supports" thrust on the slab but anchor it back, and the pull at the lower end of the anchorage is taken up by the heel slab which is loaded by the weight of earth fill acting vertically. The heel slab therefore spans horizontally between the bottoms of the counterforts, and it also is designed as a slab spanning continuously over "supports" (anchorages). The toe is designed in exactly the same manner as for the ordinary cantilever wall.

Since the wall slab and heel slab tend to tear away from the counterfort, the counterfort must be well anchored to each slab, foot by foot, by U-bars.

It was stated that the heel slab spanned between counterforts. The deflection following this action can occur freely at the rear edge of the heel, but owing to its monolithic construction with the wall slab no such deflection can occur near the junction of the two; in a similar manner the wall slab, which is free to deflect horizontally for practically its full height, is restrained by the footing at its base. There is therefore a small cantilever action between the two, and this should be provided for by a nominal amount of reinforcing steel when detailing the bars.

The exact procedure in design can best be explained by an outline example which will now be worked. The design of the members comprising the wall will also be discussed as the working proceeds.

**Example.**—Design a counterfort retaining wall of 25 ft. overall height. The equivalent fluid pressure may be taken as 30 lb. per square foot, and the bearing pressure should not exceed 4,000 lb. per square foot. Allowable stresses, steel 16,000 lb. per square inch and concrete 750 lb. per square inch.

Wall Slab.—In a number of cases the wall may from theoretical considerations alone be designed only a few inches thick. In practice, however, a counterfort wall should seldom be constructed with a wall slab thickness of less than 8 or 9 in., and many engineers would limit the minimum thickness to 12 in. Where there is any question of the quality of the concrete or workmanship, or where the maximum size of aggregate is unduly large, 12 in. should be taken as the minimum. In the present example the minimum thickness will be taken as 9 in.

The spacing of the counterforts is controlled by the maximum span suitable to the selected wall thickness; where, however, the counterfort spacing would by this method be too close to be economical (owing to the extra complication of shuttering and steel when the bays are small), the wall slab would be thickened towards the base. The batter would be supplied on the outer face where it is generally required also for appearance' sake as explained on p. 129.

The span of the wall slab is taken from centre to centre of the counterforts.

If the horizontal earth load is uniform throughout and the slab is continuous over many bays the bending moments are:

(*a*) At the supports, two-thirds of the "free" moment, or $\frac{1}{12}wl^2$.

(*b*) At mid-span, one-third of the "free" moment, or $\frac{1}{24}wl^2$.

Unless the end bay is made of a shorter span, the bending moment, owing to lack of continuity, will in this bay, and at the first interior support, be greater than these values. Owing also to possible irregularities of earth pressure the mid-span moments may in any bay be greater than $\frac{1}{24}wl^2$. It is suitable to design the support moments for $\frac{1}{12}wl^2$ and then to supply from one-half to two-thirds of the same bars or steel area at mid-span.

Assuming the most heavily stressed horizontal strip of slab to be at a depth of say 22 ft. below ground level, the horizontal earth load is $30 \times 22 = 660$ lb. per square foot. It will be remembered that the wall slab immediately above the footing is restrained by cantilever action (see p. 131). The width of counterfort in reducing either the span or load may be neglected, so that if the counterforts are spaced at $l$-ft. centres, the support bending moment in this strip 22 ft. deep is

$$\frac{660\ l^2}{12}.$$

A 9-in. wall with the horizontal bars embedded say 2 in. deep * (face of concrete to centre of bar) has an effective depth of 7 in. For stresses of 16,000 and 750 in the steel and the concrete, $p = 0{\cdot}0097$ and $R = 133{\cdot}5$. The resisting moment $Rbd^2$ is therefore $133{\cdot}5 \times 1 \times 7^2$ ft. lb. $= 6{,}550$ ft. lb. If these moments are equated we get

$$l^2 = \frac{6{,}550 \times 12}{660} = 119 = (10{\cdot}9)^2.$$

The spacing of counterforts should not be too great or the tendency will be for the wall to act more as a cantilever off the footing than as a "beam" spanning between counterforts; there is no recognised rule limiting this spacing but, generally speaking, about one-third of the overall height should not be much exceeded. In the present example we may reasonably select 8 ft. 6 in. as the maximum spacing, and this is well within the limiting span for the 9-in. thick wall.

The bending moment at the support in the wall at the 22-ft. depth is therefore

$$\tfrac{1}{12} \times 660 \times (8\tfrac{1}{2})^2 = 3{,}960 \text{ ft. lb.}$$

$$R = \frac{3{,}960}{7^2} = 81\ ;\ p = 0{\cdot}0057\ ; \text{ and}$$

$$A_s = 7 \times 12 \times 0{\cdot}0057 = 0{\cdot}48 \text{ sq. in.}$$

($f_c$ is slightly less than 550 lb. per square inch.)

Since $\frac{5}{8}$-in. bars at $7\frac{1}{2}$-in. centres are equivalent to 0·49 sq. in., these bars would be suitable. They would be supplied at the inside (or earth) face of the wall at the supports, and should extend for about one-quarter of the span clear of the counterfort in each direction and not less than "bond distance." Every third

* It is claimed by some engineers that the bars closest to the shuttering should be the vertical ones, as this assists the concrete to pack densely between bars and shuttering so providing adequate protection to the reinforcing steel. In any case a fairly liberal cover of concrete should be used.

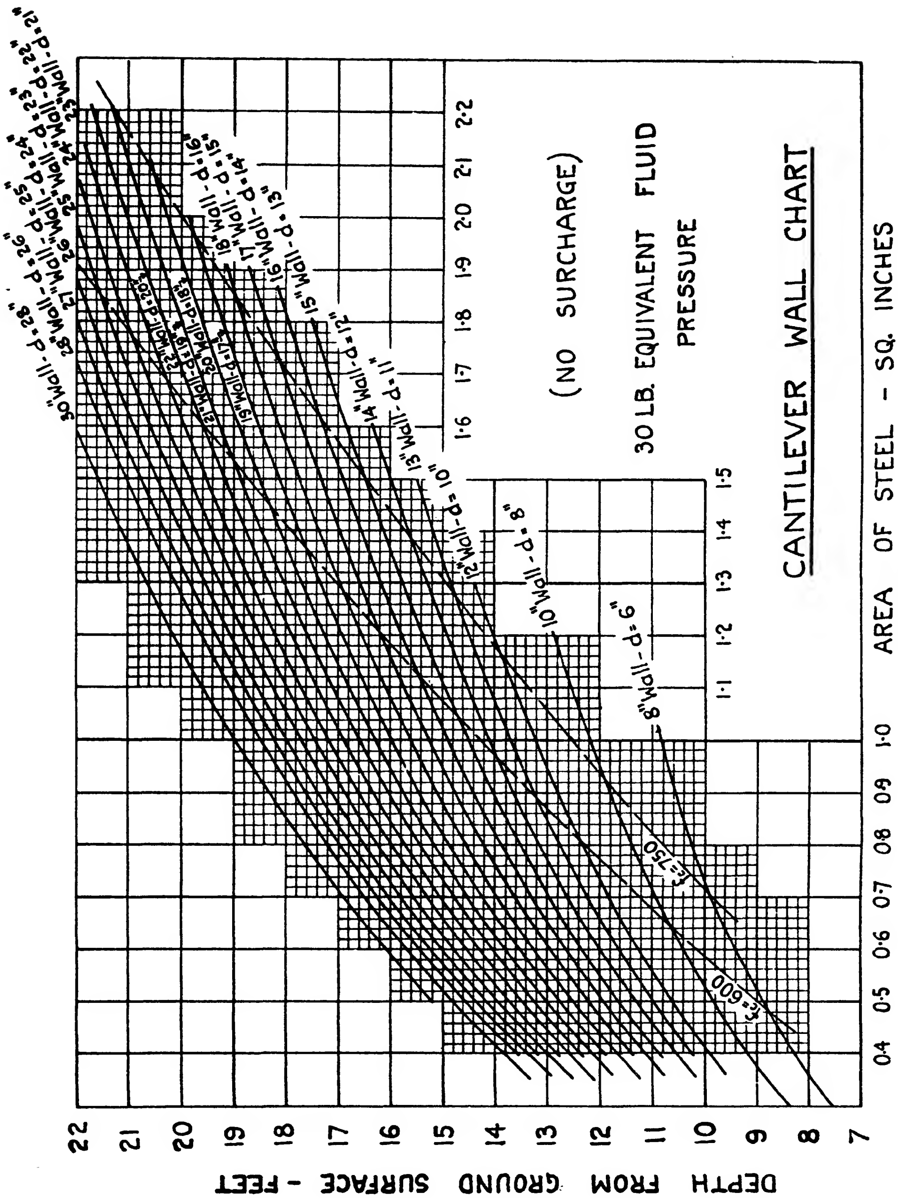

CANTILEVER WALL CHART
(NO SURCHARGE)
30 LB. EQUIVALENT FLUID PRESSURE
AREA OF STEEL - SQ. INCHES
DEPTH FROM GROUND SURFACE - FEET
0·4 0·5 0·6 0·7 0·8 0·9 1·0 1·1 1·2 1·3 1·4 1·5 1·6 1·7 1·8 1·9 2·0 2·1 2·2
7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
30" Wall-d=28"
28" Wall-d=26"
27" Wall-d=25"
26" Wall-d=24"
25" Wall-d=23"
24" Wall-d=22"
23" Wall-d=21"
18" Wall-d=16"
17" Wall-d=15"
16" Wall-d=14"
15" Wall-d=13"
14" Wall-d=12"
13" Wall-d=11"
12" Wall-d=10"
10" Wall-d=8"
8" Wall-d=6"
fc=750
fc=600

Fig. 93.

or fourth bar might reasonably be carried straight through the span in the rear face. The steel in the outer face should be about two-thirds of this amount, and, unless the same bars are used by bending them from one face to the other in the manner suggested on p. 59, suitable steel would be provided by $\frac{1}{2}$-in. bars at the same spacing. In detailing an effort should be made to relate the spacing of bars in both faces. It is often economical to run the front bars straight through in the wall slab rather than to crank them, as the handling and fixing of the bars is simpler.

The design of the counterforts will depend upon the width of the footing, and the footing may therefore be designed first.

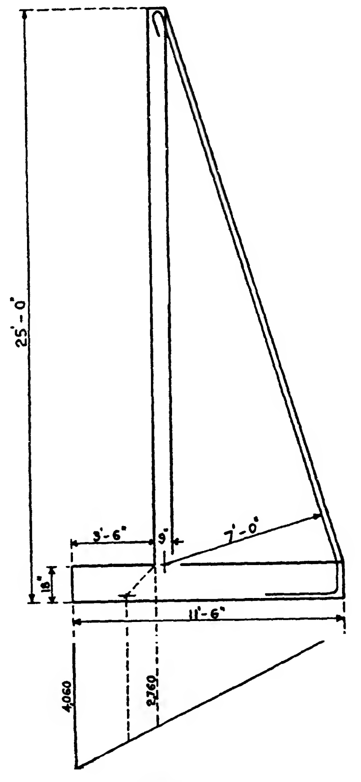



**Fig. 94.**

Footing.—The width of footing, and the position on it of the wall slab are governed by the same conditions as those operating in the case of the ordinary cantilever wall (see p. 123). $0{\cdot}45 \times 25$ ft. $= 11{\cdot}3$ ft. Try 11 ft. 6 in., with the wall face say 3 ft. 6 in. from the front edge. Assuming a footing thickness of 18 in. a section of the wall would appear as shown in *Fig.* 94.

The bearing pressures below the footing may be found by taking a slice 1 ft. wide, ignoring the counterfort. The downward loads are those due to wall slab, footing slab, and earth filling above the heel slab. The only other active force

is that due to the horizontal earth thrust which tends to overturn the wall. Moments may conveniently be taken about the rear edge of the footing base.

TABLE OF LOADS AND MOMENTS.

| Load. | | Lever arm. | Moment $A$. |
|---|---|---|---|
| $W_1$ = | $23\frac{1}{2} \times \frac{3}{4} \times 150 = 2{,}640$ | 7·625 | 20,150 |
| $W_2$ = | $11\frac{1}{2} \times 1\frac{1}{2} \times 150 = 2{,}580$ | 5·75 | 14,880 |
| $W_4$ = | $7\frac{1}{4} \times 23\frac{1}{2} \times 100 = 17{,}000$ | 3·625 | 61,700 |
| $H_e$ = | — | $\frac{30}{6} \times 25^3$ | 78,000 |
| | 22,220 lb. | 7·86 ft. | 174,730 ft. lb. |

$$\begin{array}{r} 5{\cdot}75 \\ \hline e = 2{\cdot}11 \end{array}$$

This means that the thrust cuts the base slightly in front of the middle-third, giving condition 4 referred to in *Fig.* 77. The "shortened base" is therefore $3 \times 3{\cdot}64 = 10{\cdot}92$ ft. The toe pressure is $\frac{22{,}220 \times 2}{10{\cdot}92} = 4{,}060$ lb. per square foot, and the bearing pressure diagram is as represented in *Fig.* 94. The maximum bearing pressure is only $1\frac{1}{2}$ per cent. in excess of the "allowable" pressure, and occurs only at the front edge; the bearing pressure may therefore be considered as satisfactory. (The actual conditions will depend on the considerations discussed on p. 110.)

The pressure increment along the base = 372 lb. per square foot per foot.

The pressure intensity immediately below the wall face

$= (4{,}060 - 3{\cdot}5 \times 372) = 4{,}060 - 1{,}300 = 2{,}760$ lb. per square foot.

DESIGN OF THE TOE.—The bearing pressure acting upward is split up into a uniform pressure (represented by a rectangle in the pressure diagram), and a uniformly varying pressure (represented by a triangle). The weight of the footing acting downwards may conveniently be deducted from the former.

The bending moment is calculated by multiplying the shear by the lever arm.

| | | Shear at face of wall. | Lever arm. | Bending moment at face of wall. |
|---|---|---|---|---|
| $3{\cdot}5(2{,}760 - 216)$ | = | 8,900 | $\frac{3{\cdot}5}{2}$ | 15,600 |
| $3{\cdot}5 \times \frac{1{,}300}{2}$ | = | 2,275 | $3{\cdot}5 \times \frac{2}{3}$ | 5,300 |
| | | 11,175 lb. | | 20,900 ft. lb. |

Ample embedment should be provided in footings. In the present case $d$ is taken as 15 in.

$$R = \frac{20{,}900}{15^2} = 93 \qquad \therefore p = 0{\cdot}0066 \text{ and } A_s = 1{\cdot}19 \text{ sq. in.}$$

Use $\frac{7}{8}$-in. bars at 6-in. centres ($A_s = 1{\cdot}20$; $\Sigma o = 5{\cdot}50$).

Bond Stress—

$$u = \frac{11{,}175}{5 \cdot 50 \times 0 \cdot 9 \times 15} = 150 \text{ lb. per square inch.}$$

This is high and the ends of the bars should be hooked.

Shear at the intersection of the 45-deg. plane with the bottom steel (15-in. from the wall face)

$$2 \cdot 25(3{,}225 - 216) = 6{,}750$$

$$2 \cdot 25 \times \frac{835}{2} \qquad = \underline{\phantom{0}940}$$

$$7{,}690 \text{ lb.}$$

$$v = \frac{7{,}690}{12 \times 0 \cdot 9 \times 15} = 47\tfrac{1}{2} \text{ lb. per square inch.}$$

DESIGN OF THE HEEL.—The net forces causing bending moment in the heel slab spanning between the counterforts are due to the weights of the earth filling and slab acting downwards, less the bearing pressure acting upwards. The most severe condition exists at the rear-most strip; from inspection of the pressure diagram it is clearly unnecessary to consider any upward (relieving) pressure from bearing for this strip. Here

$$216 + 2{,}350 = 2{,}566 \text{ lb. per square foot.}$$

The "free" bending moment would be:

$$\tfrac{1}{8} \times 2{,}566 \times (8\tfrac{1}{2})^2 = 23{,}200 \text{ ft. lb.}$$

Assuming that two-thirds of this bending moment are taken at the "supports," the bending moment near the counterforts is $\frac{2}{3} \times 23{,}200 = 15{,}450$ ft. lb.

$$R^* = \frac{15{,}450}{16^2} = 60 \cdot 2\,; \quad p = 0 \cdot 0042 \text{ and } A_s = 0 \cdot 81 \text{ sq. in.}$$

$\frac{3}{4}$-in. bars at 6-in. centres provide 0·88 sq. in. ($\Sigma o = 4 \cdot 71$).

The shear on this strip is

$$3\tfrac{3}{4} \times 2{,}566 = \quad 9{,}630 \text{ lb.}$$

$$\therefore\ v = \frac{9{,}630}{12 \times 0 \cdot 9 \times 16} = 56 \text{ lb. per square inch.}$$

The bond stress is

$$u = \frac{9{,}630}{4 \cdot 71 \times 0 \cdot 9 \times 16} = 142 \text{ lb. per square inch.}$$

Bars should be either bent down to supply bottom reinforcement for mid-span, alternate bars being reversed as shown in *Fig.* 95, or run straight through and hooked.

The spacing of these bars may be increased closer to the wall slab: the downward load of earth and concrete remains the same as before, but the upward bearing pressure which acts as a relief to the downward load steadily increases. The method of calculation is straightforward and need not be pursued further.

* The concrete at the top of the footing will probably have a more uniform surface than that in the base, and the embedment has been reduced by 1 in. below that provided in the base.

The bottom steel for the moment at mid-span may be supplied by bending the top bars down as already described (*Fig.* 95), or by additional bars extending straight through. The bending moment is theoretically one-half of the " sup-

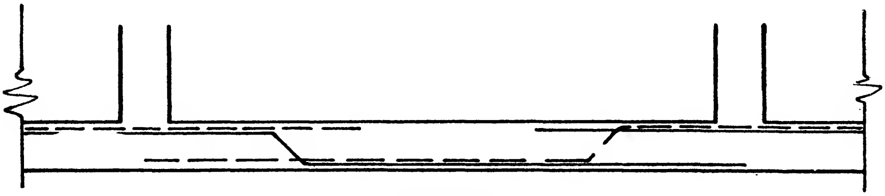


**Fig. 95.**

port " moment, but generally something more than half the same amount of reinforcement is provided.

Close to the wall slab the moment in the direction of the span between

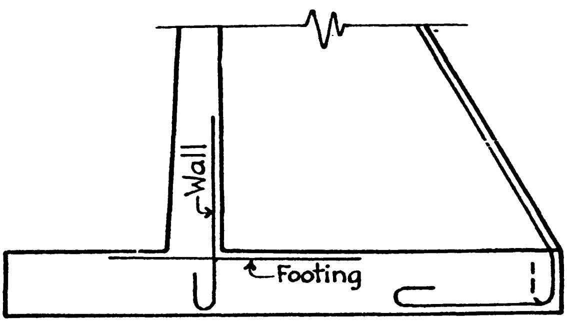



**Fig. 96.**

counterforts completely disappears owing to the stiffening effect of the wall slab which prevents deflection of the footing slab. A little cantilever reinforcement is supplied as shown in *Fig.* 96.

## Design of Counterforts.

The counterfort is a cantilever beam, fixed by the footing, and loaded by the horizontal earth pressure operating on the wall slab connecting the counterforts. The bending moment is that due to the thrust on a length of wall equal to one bay. Fixity is provided by the heel slab loaded by the earth fill which it tends to lift.

The maximum moment in the counterfort occurs at the level of the footing top, and is :

$$8\tfrac{1}{2} \times \frac{30}{6} \times (23\tfrac{1}{2})^3 = 553{,}000 \text{ ft. lb.}$$

It may be assumed that the centre of compression in the " flange " (wall slab) is at the middle of the wall (actually it will be slightly nearer the front

face), and the " lever arm " from this point to the steel is 7 ft. (measured normal to the steel: see discussion on p. 89) (see *Fig.* 94).

$$\therefore A_s = \frac{553{,}000}{7 \times 16{,}000} = 4{\cdot}94 \text{ sq. in.}$$

Four $1\frac{1}{4}$-in. bars have an area of 4·91 sq. in.

Both bending moment and " lever arm " decrease towards the top of the counterfort, but the former much more rapidly. Two of these bars may therefore be stopped off some distance below the top. Frequently they are merely hooked, but in the author's opinion it is preferable to bend them forward and anchor them into the slab as shown in *Fig.* 97. The bending moment should be checked at

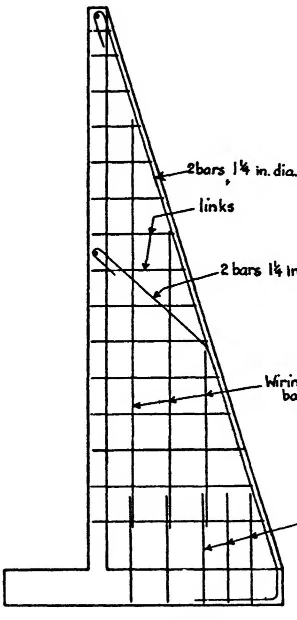



**Fig. 97.**

the point where the bars are bent away or stopped off. The pair of bars carried to the top of the counterfort should be well anchored to the slab, and bars should be threaded through the hook.

It remains in detailing to " hang up " the heel slab to the counterfort, and to anchor the wall slab securely to the counterfort. This is done in the former case by U-bars and in the latter case by links which are carried round the main counterfort reinforcement; both details are shown in *Fig.* 98. Wiring bars are also added as shown in *Fig.* 97.

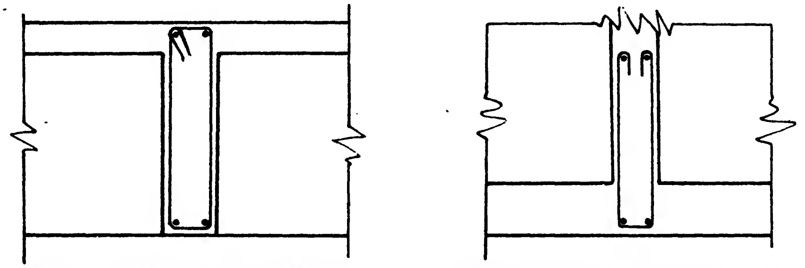
The worst position for the heel slab is at the back strip where the earth and concrete load which the counterfort tends to lift is equal to twice the shear in the strip (that is, the shear from the two sides of the counterfort), or

$$2 \times 9{,}630 = 19{,}260 \text{ lb.}$$

The **U**-bars are in direct tension and, if the steel stress is 14,000 lb. per square inch, the required steel area is $A_s = \dfrac{19{,}260}{14{,}000} = 1{\cdot}375$ sq. in. per foot. The counterfort bars more than supply this at the rear edge, and closer towards the wall the requirement is steadily reduced owing to the increasing bearing pressure.

Typical details are shown in *Figs*. 95, 96, 97, and 98.

There are a few general points to be considered in designing these walls.



SECTIONAL PLAN THROUGH COUNTERFORT AND WALL SLAB. SECTIONAL ELEVATION THROUGH COUNTERFORT AND FOOTING SLAB.

**Fig. 98.**

(1) The height at which the ordinary cantilever wall gives place to the counterfort wall: this question will receive a different answer in varying localities. The reduced quantity of concrete in the counterfort wall must be balanced against the increased cost and complication of shuttering and steel placing, and where skilled labour is not obtainable the ordinary cantilever wall is often built to a height of from 25 to 30 ft. In favourable circumstances 18 or 20 ft. may be the dividing line. It should also be borne in mind, where foundations are doubtful until opened up, that the cantilever wall is more easily adapted to increases in height after the design has been prepared.

(2) The wall slab in the counterfort wall is designed as continuous. There is lack of fixity at expansion joints or at the end of the wall, and the end bay should therefore be designed for a larger bending moment both at mid-span and at the first interior support. An alternative is to construct the end bay with a slightly shorter span.

(3) Expansion joints are not so easily provided as in the cantilever wall. The same form of joint (see *Fig*. 92) may be provided if short lengths of wall

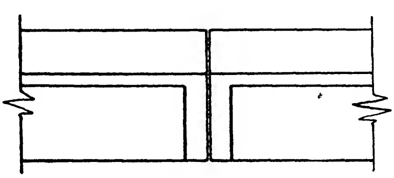


**Fig. 99.**

slabs are cantilevered beyond counterforts so as to butt against each other. The more common form of joint, however, is made by constructing two counterforts close together as shown in *Fig*. 99, some simple form of key being provided between them after the manner of *Fig*. 92. These joints are usually a little farther apart

than in the simpler type of wall, but should generally be spaced not farther apart than about 40 to 50 ft.

In very high walls it may become economical to construct a small buttress to the toe slab at the position of the counterfort; such a buttress would run from the front edge of the toe at an angle of about 45 deg. as shown in *Fig.* 100,

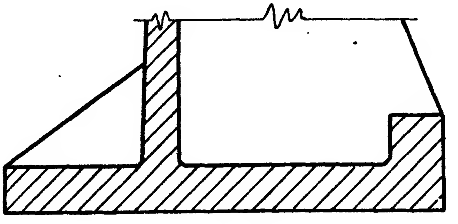


**Fig. 100.**

and the toe slab would span longitudinally similarly to the heel slab. A further variation, also shown in *Fig.* 100, is in the provision of a heel beam spanning between the counterforts: with this modification the heel slab spans between the wall slab and the heel beam. Provision should be made for resisting the shear stress (diagonal tension) in the heel beam which is likely to be high.

Provision for drainage of the back filling by weep holes must be made in the counterfort type of wall in exactly the same way as for other retaining walls.

It is sometimes necessary to provide some form of relief to the plainness of a large surface area in a retaining wall. The "construction" or expansion joints provide a slight relief. A coping may quite easily be constructed as shown in section in *Fig.* 101. A last resort is to shutter portions of the wall slab to form

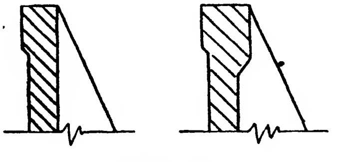


**Fig. 101.**

sunk panels, but this is not economical: if this method is adopted care should be taken that the concrete cover on the reinforcing steel in the wall slab is not encroached upon or rust stains may appear on the face; these are far more destructive of appearance than plainness, and are also of structural significance.

# CHAPTER XIII

## PRESTRESSED REINFORCED CONCRETE

SINCE the prestressing of reinforced concrete is now widely established the student should be familiar with the fundamental principles underlying this practice.

The basic idea of prestressing is to produce a member which, before the external loads it is designed to carry have been applied, is by the manner of its construction already in a state of internal stress; these stresses are, however, of the opposite sense to the stresses which will result from the application of dead and live loads, and are thus intended largely to cancel them. Pre-induced compression will be neutralised by subsequent tension, and vice versa. A simple example is the cylindrical tank. When a tank is filled with liquid the fluid pressure tends to burst the tank, subjecting the circular wall to direct tensile stress (termed "hoop" tension). If steel bands are placed around the outside and, while the tank is still empty, the bands are tightened (that is, are put in a state of tension) the circular wall will be compressed (that is, put in a state of direct compression). If the tension of the outer bands is exactly equal to the tension which, without the help of the steel bands, would have been produced in the circular wall by the pressure of the liquid when the tank is filled, the wall will be free from stress. In practice the stresses are never so accurately calculated as to result in a completely unstressed member, but an approximate balance will leave the member with low stresses only.

Since concrete is weak in tension, and cracking occurs at comparatively low stress, it is desirable to avoid tension. The wall of a reinforced concrete cylindrical tank as normally designed is one example of a structure in which tension in the concrete cannot be avoided. The commonest example is the tension side of a beam. Although in most buildings the degree of cracking accompanying normal design is of no real consequence, in exposed positions cracking may be harmful, and in any structure the reduction of cracking to a minimum is desirable. It has been explained how precompression in the wall of a tank can eliminate or reduce tensile stress and so avoid the risk of cracks. The same principle can be applied to a beam or any other member.

In the case of a simply-supported beam the lower portion is placed in a state of compression. Subsequent loading will not produce tension (as would normally be the case) until after this precompression has been neutralised. The extent of the prestressing can be controlled and related to the subsequent loading so as to eliminate tension in the concrete altogether, or to limit it to any desired amount. It is characteristic of prestressed reinforced concrete beams that, even if they are overloaded to the stage where cracking occurs, removal of the overload will be followed by closing of the cracks so long as the steel has not been

stressed beyond its yield point; this is not necessarily so in the case of ordinary reinforced concrete beams. In normal reinforced concrete beams cracking of the concrete below the neutral plane makes it necessary in design to neglect the area below the neutral axis; in prestressed beams the whole cross-sectional area can be taken into account, as no cracking occurs. This consideration also permits the use of a modular ratio of, say, 8 instead of the usual 15; it also substantially reduces the principal tensile stresses due to shear forces.

There are two main methods by which reinforced concrete beams are prestressed. In both methods special high-tensile steel wires are used.

Method No. 1.—The reinforcement, placed in position within the moulds, is stretched by jacks thrusting against specially-constructed abutments; in some cases the moulds may be used for the purpose, but often something much more substantial is required as the load on the jacks may be very high. The concrete is then placed, embedding the stretched wires which are kept in tension by the jacks until the concrete has attained sufficient strength to withstand the high compressive and bond stresses which will result when the jacks are released. When this externally-applied load is relaxed the steel is prevented from returning to its unstretched condition only by the concrete, which is thereby compressed; a balance is set up, the total compression in the concrete being equal to the total tension remaining in the steel. The steel is generally placed in the lower part of the beam, and this eccentric condition introduces bending in the beam; if the steel is below the "middle third" the top of the beam will be put in a state of tension, and it may be necessary to have reinforcing bars in the top of the beam.

Method No. 2.—The reinforcement, usually in the form of a cable of wires, is encased in a sheath which prevents adhesion between the concrete and the steel. The concrete is cast in the mould and around the sheath containing the steel. When the concrete has sufficiently hardened the steel is stretched, the reaction being taken on the concrete, and a distribution of stress similar in the main to that obtaining in Method No. 1 results. There is, however, a small and generally unimportant difference. In Method No. 1 the reaction between the concrete and the steel is produced by bond stress which is distributed along the bar, and the stresses in the concrete and the steel are therefore built up from zero at the ends to a maximum at midspan; this corresponds roughly, but with reverse sign, to the stress in the lower half of the concrete which would result in the case of a simply-supported beam under load. In Method No. 2 the reaction between the concrete and the steel occurs in full at the ends of the beam, and the same condition obtains throughout.

The stress distribution in the concrete over the cross-sectional area in a prestressed reinforced concrete beam is illustrated in *Fig.* 102. The gross (or "equivalent") cross-sectional area is taken as $A$, and the equivalent moment of inertia as $I$. The residual total tensile force in the steel group is taken as $F$, acting at a distance $e$ from the centroid. The extreme fibre stresses for the beam as constructed are obtained by application of equation (44), p. 97. Thus the maximum compressive stress in the concrete,

$$f_{c1} = \frac{F}{A} + \frac{Fey_1}{I} \quad . \quad . \quad . \quad . \quad . \quad (66)$$

and the stress at the top of the beam, if in compression as in (b),

$$f_{c2} = \frac{F}{A} - \frac{Fey_2}{I} \quad . \quad . \quad . \quad . \quad . \quad (67)$$

If the magnitude of the bending moment introduced by the eccentricity of force $F$ is such as to introduce tension in the top face as shown in $(b_1)$,

$$f_{t2} = \frac{Fey_2}{I} - \frac{F}{A} \quad . \quad . \quad . \quad . \quad . \quad (68)$$

These equations give the concrete stresses for the beam in its unloaded condition, no account having been taken of dead or live loads. The stress distribution resulting from a bending moment $M$ (representing, say, the effect of dead and live loads) is illustrated at (c). This ignores the prestressing force and assumes that the concrete is uncracked. Condition (c) can then be superimposed on condition (b) or $(b_1)$ to give the combined resulting stress distribution.

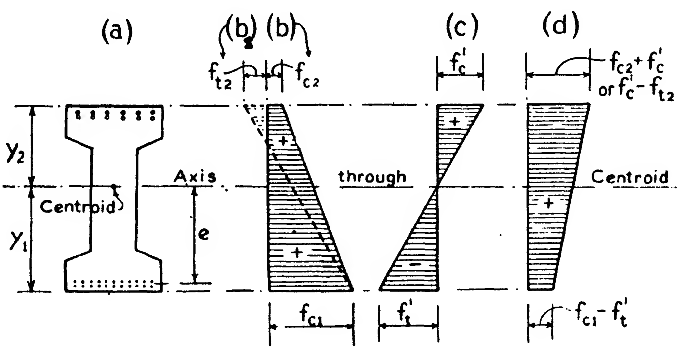



**Fig. 102.—Distribution of Stresses in a Prestressed Concrete Beam.**

At (c) the maximum stresses are $f_t' = \dfrac{My_1}{I}$ (tension) . . . (69)

and $$f_c' = \frac{My_2}{I} \text{ (compression)} \quad . \quad . \quad . \quad . \quad (70)$$

The resulting working stresses are shown at (d), which is simply the algebraical combination of (b) and (c).

Selection of the values of $F$, $A$, and $I$ is the purpose of design, and these may be chosen so as to produce compression over the whole of the section as at (b), or a small permissible tension in one face as at $(b_1)$. The maximum compressive stress in the concrete, $f_{c1} - f_t'$ should not exceed the safe compressive strength of the concrete, or if this value is negative the numerical amount should be well within the known tensile strength of the concrete.

Similarly for $(f_{c2} + f_c')$ or $(f_c' - f_{t2})$.

The value of $F$ to be taken in design as the net effective stretching force is less than the total load applied through the jacks. Several factors account

for the loss. In Method No. 1 the initial load which stretches the wires is taken on the abutments or other external structure provided for this purpose independently of the beam. After the beam has gained sufficient strength the load is transferred to the beam, equilibrium between the steel and the concrete being set up through bond action between the two. There will thus be an elastic shortening of the beam as it takes up compression, and the steel, of course, shortens to exactly the same extent (assuming that there is no " slip " or bond failure); this shortening of the steel results in a falling off in the prestressing load, stress being assumed proportional to strain. Method No. 2 does not involve this particular loss as the stretching of the steel is done initially by thrusting against the beam itself and a balance is maintained from the first. In both methods shrinkage of the concrete with the lapse of time results in a corresponding shortening of the steel with consequent loss of stress, and creep or plastic yield in both concrete and steel results in a further reduction of the elastic strains with a corresponding loss of prestress. These losses are quantitatively determinable and therefore a net value can be deduced for $F$.

In Method No. 1, at the time of transferring the load from the abutment to the beam the loss of prestress (intensity) is $n$ times the stress which is then set up in the surrounding concrete.

If $P$ = initial load taken on the abutments, and $P_1$ = load taken up by compression in the beam on transfer, then the loss of load in the process of transfer will be

$$P - P_1 = P_p \text{ (say)} \quad . \quad . \quad . \quad . \quad (71)$$

If $A_p$ is the combined cross-sectional area of the wires, and $f_p$ is the stress in the concrete immediately surrounding the steel, then $P_p = A_p n f_p$.

By equation (44),
$$f_p = \frac{P_1}{A} + \frac{P_1 e^2}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

Solving these equations,

$$P_p = \frac{P}{1 + \dfrac{1}{A_p n\left(\dfrac{1}{A} + \dfrac{e^2}{I}\right)}}.$$

The shrinkage coefficient of concrete $\varepsilon$ may vary between 0·0002 and 0·0005. Taking the worst case, as required by good design, the estimated loss of stress in the stretched steel from this cause will be 0·0005 $E_s$. The maximum loss due to creep or plastic yield in the concrete may reasonably be considered equal to the shrinkage loss, so bringing the total loss from these two causes to 0·001 $E_s$, which is, say, approximately 30,000 lb. per square inch. This loss of stress operating over an area of steel $A_p$ gives a reduction $P_c$ in load of 30,000 $A_p$. Stating this in general terms,

$$P_c = 2\varepsilon . E_s . A_p \quad . \quad . \quad . \quad . \quad . \quad (73)$$

Hard cold-drawn steel wire is used in prestressed beams, and this is stressed initially to from 120,000 to 140,000 lb. per square inch. Such a condition will result in plastic yield in the steel involving a loss of load, $P_s$, of anything up to 7½ per cent. of the initially effective load. $P_s$ may therefore be taken as equal to 0·075$F$.

The ultimate net stretching force $F$ thus becomes equal to $P - P_p - P_c - P_s$. With Method No. 2, $P_p$ is, of course, zero since there is no transference of load from the abutments to the beam.

Ordinary mild steel reinforcement is unsuitable for prestressed beams for several reasons, the main ones being: (1) The quantity necessary to provide the total load could not readily be accommodated within the compass of the beam; (2) the low yield point; (3) the proportionate losses due to $P_p$, $P_c$ and $P_s$; and (4) the necessity of providing a margin of stress for an increase in tension in the steel under working conditions. Steel as described is therefore generally used in the form of a large number of small wires of about $\frac{3}{16}$ in. diameter, having a high ultimate strength somewhat above 200,000 lb. per square inch, with a yield stress of about 160,000 lb. per square inch. With such stresses the various losses referred to represent only a small proportion of the high initial prestress, but this would be very different if mild steel were employed.

Besides providing freedom from cracking, prestressed construction enables the use of very shallow beams with much saving of dead load, making possible longer spans. Use has been made of the system in girder and arch bridge construction, liquid containers, railway sleepers, and other precast or independent units. Difficulties have not yet been entirely overcome in the case of continuous beams and framed structures. The methods have been described only broadly, and in practice there are many variations in detail and in the mechanical equipment employed. In the case of Method No. 1, if large numbers of units are required, the wires are often stretched over a very long length, and the units are formed end to end like a train, each beam corresponding to a coach; the wires are then snipped between the ends of beams after the load has been released from the abutments.

The stress distribution is complex, the balanced loads so high, and the beams so slender that handling of these precast members is a delicate operation. Handling and storing of prestressed members should be undertaken only by those who understand thoroughly the characteristics of the units.

# GLOSSARY OF TERMS

(Note.—The following descriptions are not intended as strict dictionary definitions, but are given as simple explanations of the general meaning of the words to which they apply.)

ANGLE OF FRICTION (referring to soils).—The limiting angle at which grains of a material (free from cohesion) will just slide over each other under the force of gravity; for practical purposes equal to the angle of repose.

ANGLE OF REPOSE.—The maximum angle at which a heap of granular material, free from cohesion, will stand.

ASYMPTOTIC (Asymptote).—The condition when a curve converges upon a straight line so as to become tangential with it at infinity.

AXIAL.—Along the axis.

AXIS.—A principal line of reference in a body or area, generally symmetrically placed.

BATTERED (Batter) (p. 129).—Sloping with the vertical.

BENDING MOMENT (p. 3).—The effect of externally applied forces which produce bending in a member.

BOND "DISTANCE" (p. 51).—The length of bar theoretically required to develop by bond stress the tensile working stress in the bar.

BOND STRESS (p. 35).—The stress set up between a reinforcing bar and the surrounding concrete, due to adhesion and friction, which prevents relative slip between them.

BRIDGE SEAT (p. 122).—That area on the substructure (abutment or pier) of a bridge on which the superstructure rests.

CANTILEVER (p. 2).—A beam supported solely by being fixed at one end.

CENTRE OF GRAVITY (p. 39).—The point in a body or section about which, if suspended, the body will in all positions balance.

CENTRE OF PRESSURE (p. 117).—The point in a surface through which the resultant of the applied pressures acts.

COHESION (p. 110).—The binding force, apart from gravity, which unites the individual particles of a body or mass.

CORE AREA (p. 94).—The cross-sectional area of a reinforced section (usually in compression) included inside the links, omitting the concrete cover.

COVER.—The concrete on the outer side of the reinforcement employed to afford bond and to protect the steel.

CRIMP, OR CRANK (p. 96).—Applied to a reinforcing bar, referring to a short straight offset or double bend.

CRITICAL SECTION.—A section of maximum stress, and one which therefore has a controlling influence on the design.

CUSP.—Any point in a curve where an angle (sharp) is formed.

DEAD LOAD (p. 24).—The load of the member or structure itself, including any permanently applied loads which contribute to an unchanging state of stress.

DIAGONAL TENSION (p. 39).—Tensile stresses set up diagonally in a reinforced concrete beam due to a combination of shear stresses with the primary tensile stresses.

ECCENTRICITY (Eccentric) (p. 92).—The distance by which a force is out of centre, or out of line with the axis of the body to which it is applied.

EFFECTIVE DEPTH (pp. 40, 53).—In a reinforced concrete section subjected to bending: distance from the compressive face of the concrete to the centre of gravity of the tensile reinforcement.

EFFECTIVE SPAN (p. 53).—The span length effective in controlling the magnitude of the bending moment due to transverse load.

ELASTICITY (Elastic) (p. 8).—The capacity for strain accompanied by stress which enables a member to return to its original state when the load is removed.

EMBEDMENT.—The distance at which a reinforcing bar is embedded from the face of the concrete, measured to the centre of the bar.

EQUILIBRIUM (p. 3).—The state of having all forces balanced so that the body or structure is at rest.

FACTOR OF SAFETY (p. 38).—The ratio between the load or stress at which a material, member, or structure will fail and the load which is imposed on it under working conditions.

FINAL SET (p. 30).—The condition, subsequent to the initial set, when concrete just becomes firm, prior to commencing the hardening process.

FLANGE (p. 71).—Applied to a T-beam: the slab in a T-beam which acts as the compression member.

FLEXURAL (Flexure) (p. 14).—A term which denotes bending (generally with a suggestion of comparative freedom—as "flexible"): thus "flexural stress" means "bending stress."

"FREE" BENDING MOMENT (pp. 15, 18, 22).—Applied to continuous spans: the bending moment which would exist in a freely-supported span of the same length and under the same loading.

GRANULAR AND NON-COHESIVE.—The state of a body in which the individual particles are free to slide over each other, hindered only by gravity and the frictional forces inherent in the material.

**Gravity Axis** (p. 98).—A line passing through the centres of gravity of successive strips forming a sectional area.

**Hard-pan** (p. 107).—Coarse sand or gravel cemented firmly with a small amount of clay which renders it hard.

**Homogeneous** (p. 13).—Term applied to a material to imply the state of having exactly similar physical properties in all directions.

**Impact Factor** (p. 24).—A factor applied to the live load to make it equal to a static load which would produce the same state of stress in the member to which the load is applied.

**Influence Line** (p. 25).—A curve drawn for any one section in a system to show by its vertical ordinate the effect on that section of a unit load placed successively at all other points in the system.

**Initial Set** (p. 30).—The condition after mixing when concrete first begins to lose its plasticity.

**Initial Stress** (p. 38).—Stress set up in a member during erection or construction due to causes other than the normally applied dead or live load, and which remain (e.g. stress in a reinforcing bar set up by shrinkage of the concrete).

**Lever Arm.**—The perpendicular distance from a force to the centre about which moments are taken.

**Live Load.**—A moving or variable applied load, as distinct from dead load.

**Middle Third** (p. 96).—The middle one-third of a section, inside which area an applied thrust must pass if the whole section is to be in compression.

**Modular Ratio** (p. 37).—The ratio of the modulus of elasticity of reinforcing steel to that of concrete.

**Modulus of Elasticity** (p. 8).—The ratio of stress to strain. The value for any material is thus the unit stress required to produce unit deformation in unit length.

**Moment** (p. 3).—The turning effect of a force about a point or axis, measured by the product of the force and the lever arm.

**Moment of Inertia** (p. 10).—The term used to denote the property of a sectional area which is measured by the sum of the products of the elementary portions of the area and the squares of their distances from a given axis; a mathematical function represented by $\Sigma ar^2$.

**Moment of Resistance** (pp. 10, 39).—The moment developed (usually in a section) by the internal stresses to resist an applied bending moment.

**Neutral Axis** (p. 9).—The line or axis in a cross section where the neutral plane intersects the plane of the section.

**Neutral Plane.**—The plane in a member subjected to bending which separates the compressed fibres from those which are stretched. The fibres lying in the neutral plane are neither in compression nor tension, and remain unchanged in length after bending takes place.

**Plasticity** (Plastic) (p. 34).—The capacity for deformation under load, unaccompanied by stress (as opposed to "elastic").

**Point of Inflexion** (p. 16).—A point in the bending moment diagram where the bending moment has zero value as it changes its algebraic sign.

**Prestressing** (p. 141).—The action of inducing in an unloaded member stresses of a contrary nature to those due to the loading and thereby relieving the latter stresses.

**Puddling** (Puddle) (p. 118).—To consolidate a fill by repeated saturation of water, commencing the saturation from the bottom (by means of pipes or tubes which penetrate to the bottom of the fill) and proceeding upwards to the top surface.

**Punching Shear** (p. 105).—The shear developed in a slab or base (such as a column footing) by a load applied over a limited area, tending to allow the portion of the slab to which the load is directly applied to punch through the remainder of the slab.

**Reaction.**—A passive force developed by (or "reacting" to) an applied or active force.

**Resolution of Forces.**—The replacing of a force or forces by other forces acting in different directions but which together have an equivalent effect.

**Resultant.**—A single force which in magnitude and direction is capable in effect of exactly replacing two or more other forces.

**Shear** (pp. 4, 11).—The force which tends to slide the two portions of a transversely loaded beam on each side of a section across each other in a direction parallel to the applied loads.

**Slip** (p. 38).—Relative movement between a reinforcing bar and the surrounding concrete, signifying bond failure.

**Static Load** (p. 24).—A stationary load.

**Stem.**—Applied to the cross-section of a T-beam, and referring to the portion below the flange (see also "Web").

**Strain** (p. 8).—Deformation under load, accompanied by stress (usually given for a unit length).

**Stress** (p. 8).—A passive internal force reacting to, and resisting the applied forces, and attending strain.

**Strike** (p. 36).—Term applied to the removal of falsework.

**Stringer.**—A longitudinal beam, usually secondary to the main beams.

**Stripping** (Strip) (p. 35).—Term applied to the removal of forms from the concrete face.

**Surcharge.**—A load applied from above the level of the top of a retaining wall to the retained earth, and which tends to increase the side pressure; due, say, to an overburden of earth, or to a structure, or to live load.

**Symmetry** (p. 18).—The condition of perfect balance about a central axis of the halves of a body or diagram, so that one half is the image of the other.

**Unit Stress.**—Stress per unit of area. The word "stress" used alone, strictly speaking, has the same meaning, but the axial load in a member is sometimes referred to as "total stress."

**Unsupported Length** (p. 93).—The maximum length of a compression member in which no lateral support is supplied externally.

**Web** (p. 47).—The portion of a T-beam below the flange, as seen in elevation. A cross section through the web would be referred to as the "stem."

**Working Stress** (p. 39).—General meaning: the maximum allowable stress in a member as ruled by the specification governing the design. Occasional meaning: the stress actually existing in a loaded member.

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